Aggregate Demand and Irreversible Investment

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Motivation

To what extent micro frictions matter for macro dynamics? (with financial frictions and (partial) irreversibility)

When we consider the impact on individual firms' investment and financing decisions

- Theoretically, they have been studied extensively in the literature
- Empirically. they have been shown to match micro data

When we consider the impact on aggregate variables in response to first-moment shocks

- Each has been found to generate relatively little amplification
- It requires financial shocks with financial frictions or uncertainty shocks with irreversibility

This paper studies the propagation of first-moment aggregate shocks through the interaction between financial frictions and the partial irreversibility of fixed capital and policies to mitigate recessions

We find this combination delivers substantial amplification and counter-cyclical fiscal policy is beneficial

Road Map

Model

- Heterogenous firms
- Partially irreversible fixed capital and financing constraint on working capital
- Idiosyncratic prod shocks and endogenous default risks
- Monopolistic competition

Model Validation and Calibration

- Characterize the decision rules and test that with micro data
- Calibrate the model to the Chinese economy (manufacturing sector)

Quantitative Results

- Study transition dynamics in response to aggregate productivity shocks
- Fit in counter-cyclical fiscal policy and study impacts on welfare

Preview of Results

Mechanisms of Model

- Trade off: sell k at a costly price with lower default risk and option value versus keep k with higher default risk and higher option value
- Partially Irreversible fixed capital: Limits cash flow from liquidation & tightens financing constraint
- Unbalanced production $\Rightarrow$ Higher default risks and risk premium $\Rightarrow$ Misallocation, inefficient liquidation, and default
- Output loss $\Rightarrow$ Aggregate demand externality $\Rightarrow$ "Quasi aggregate productivity shock"

Quantitative Results: 2 percent negative aggregate shock will lead to

- 3.9% $\downarrow$ in output, 2% $\uparrow$ in dispersion in ARPK, 20% $\uparrow$ in default probability
- The maximum drop in output is closed to 2 percent if we shut down one of the frictions
- "Aggregate demand externality" accounts for 1/3 of amplification
- A modest counter-cyclical policy is welfare-improving: increase G by 2.1 percent when Y drops by 1 percent

Related Literature

Propagation and amplification of aggregate shock through micro frictions

- Kiyotaki Moore (1997) Cooley Quadrini (2001) Kocherlakota et al. (2000) Khan and Thomas (2013); Abel and Eberly (1994) Abel and Eberly (1996) Bertola and Caballero (1994) Baley and Blanco (2022) Thomas (2002) Khan and Thomas (2008) House (2014) Winberry (2018) Ottonello (2018) Kermani and Ma (2023) Cooper and Haltiwanger (2006); Bloom (2009) Bloom et al. (2007); Eisfeldt and Rampini (2006) Lanteri (2018) Chen et al. (2023); Caggese (2007) Cui(2022)

Models with financial heterogeneity and endogenous default risk

- Khan et al. (2014) Ottonello and Winberry (2020) Guntin (2022) Bloom et al. (2018) Gilchrist et al. (2014) Arellano et al. (2019)

Misallocation

- Restuccia and Rogerson (2008) Hsieh and Klenow (2009); Moll (2014) Midrigan and Xu (2014); Buera et al. (2011) Buera and Shin (2013) Gopinath et al. (2017) Baqaee and Farhi (2020); Sandleris and Wright (2014) Oberfield (2013).

Distributional effect of macroeconomic policies

- Blanchard Kiyotaki (1987) Farhi and Werning (2016) Kaplan et al. (2018) Lanteri and Rampini (2023). Baqaee et al. (2023)

Outline

- **Model**
- Model Characterization
- Empirical Evidence
- Calibration
- Quantitative Results

Model Environment

Time is discrete and infinite. No aggregate uncertainty.

Goods

- Intermediate firms produce differentiated goods under monopolistic competition
- Final good producer aggregate differentiated goods into final good under perfect competition
- Final good can be consumed or used to produce capital goods
- Firms purchase capital at the price q_t and sell at ηq_t

Hosueholds

- Representative "Entrepreneur Family" with a measure of $1 - M$
  - consists of a continuum of family members
  - each member runs an intermediary firm with initial equity transfer from family
  - firms use internal funds and external borrowing to finance investment
  - pay non-negative dividends and consume together
- "Worker Family" with a measure of M, hand to mouth

Bank

- Raise funds in one-period risk-free bond and make one-period defaultable loans

Technology

Production technology for intermediate firms

$$y_{it} = \underbrace{A_t}_{\text{Aggregate Productivity}} \underbrace{z_{it}}_{\text{Persistent Component}} \underbrace{\varepsilon_{it}}_{\text{Transitory Component}} k_{it}^{\alpha} \ell_{it}^{1-\alpha}$$

$$\log z_{i,t} = \rho \log z_{i,t-1} + \nu_{it}, \quad \nu_{it} \sim N(0, \sigma_\nu^2), \quad \log \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$$

The final good is a CES aggregate of intermediate good and we normalize the price of final good to be one

$$Y_t = (\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di)^{\frac{\theta}{\theta-1}}, \quad P_t = (\int_0^1 p_{it}^{1-\theta} di)^{\frac{1}{1-\theta}} = 1$$

Under monopolistic competition, intermediary firms face a downward-sloping demand curve and make decisions taking aggregate output as given:

$$\frac{y_{it}}{Y_t} = (\frac{p_{it}}{P_t})^{-\theta}$$

Then the revenue for firm i in terms of final good can be written as

$$\frac{p_{it} y_{it}}{P_t} = (A_t z_{it} \varepsilon_{it} k_{it}^{\alpha} \ell_{it}^{1-\alpha})^{1-\frac{1}{\theta}} \times Y_t^{\frac{1}{\theta}} \tag{1}$$

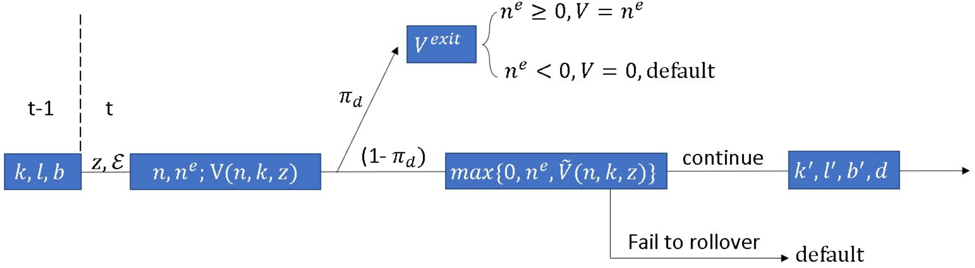
Intermediate Firms' Problem: Timeline

The firm i's objective is to maximize the expected discounted value of dividends.

$$V_{i0} = \mathbb{E}_0 \sum_{t \geq 0} \Lambda_t d_{it} \tag{2}$$

At the beginning of period t.

1. Firm i inherit k_{it}, l_{it}, b_{it}. Individual prod is realized.
2. With prob π_d, firms receive exogenous death shock. Choose default or not.
3. With prob $1 - \pi_d$. Choose between default, exit, or continue.
4. Continuing firms choose $d_{i,t+1}, k_{i,t+1}, \ell_{i,t+1}, b_{i,t+1}$



Intermediate Firms' Problem: With Death Shock

Liquidation value

$$n^e = (Az\varepsilon k^\alpha \ell^{1-\alpha})^{1-\frac{1}{\theta}} Y^{\frac{1}{\theta}} + (1-\delta)\eta q k - b \tag{3}$$

Book value of net worth

$$n = (Az\varepsilon k^\alpha \ell^{1-\alpha})^{1-\frac{1}{\theta}} Y^{\frac{1}{\theta}} + (1-\delta) q k - b \tag{4}$$

Firms choose between

- default without finishing production, pay zero dividend
- finish production, repay debt and pay non-negative dividends

$$V^{exit}(n,k,z) = \max\{n^e, 0\} = \max\{n - (1-\delta)(1-\eta)qk, 0\} \tag{5}$$

Let us define $V(n,k,z)$ as the firm value before receiving the death shock but after receiving the productivity shock

$$V(n,k,z) = \pi_d V^{exit}(n,k,z) + (1-\pi_d)\max\{V^{exit}(n,k,z), \tilde{V}(n,k,z), 0\} \tag{6}$$

Intermediate Firms' Problem: Continuing Firms

Continuation Value

$$\tilde{V}(n,k,z) = \max_{d,k',b',l'} d + \mathbb{E}[\Lambda' V(n',k',z')|z] \tag{7}$$

Subject to:

$$d = \underbrace{n-(1-\delta)qk}_{\text{liquid wealth}} + \underbrace{F(b',k',l',z)}_{\text{external borrowing}} - \xi - w\ell' - q(k'-(1-\delta)k)\times\mathbb{I}_{k'>(1-\delta)k} + \eta q((1-\delta)k-k')\times\mathbb{I}_{k'<(1-\delta)k} \tag{8}$$

$$d \geq 0 \tag{9}$$

$F(b',k',l',z)$ is endogenous, taking default risks into account. This can be solved with the default threshold simultaneously

Default Threshold For Continuing firms

Not all continuation firms' problems yield a solution since some of them can't satisfy non-negative dividend payment constraints no matter what choices are made.

We can define default threshold $\underline{n}(z,k)$ as the minimum starting net worth such that the maximum of dividends payment one can make is zero.

$$\max_{b',k',l'} \{\underline{n}(z,k)+F(b',k',l',z)-wl'-\xi-qk'\times\mathbb{I}_{k'>(1-\delta)k}+\eta q((1-\delta)k-k')\times\mathbb{I}_{k'<(1-\delta)k}-(1-\delta)qk\} = 0$$

Rearrange:

$$\underline{n}(z,k) \equiv -\max_{b',k',l'} F(b',k',l',z)-wl'-\xi-qk'\times\mathbb{I}_{k'>(1-\delta)k}+\eta q((1-\delta)k-k')\times\mathbb{I}_{k'<(1-\delta)k}-(1-\delta)qk \tag{10}$$

Proposition 1

Bank and Debt Price

- The bank is owned by the representative entrepreneur family and risk neutral
- It raises funds by issuing one-period risk-free bonds and making one-period non-state contingent defaultable loan

In the case of default, the bank can recover:

$$\min(b', (1-\tau^D)(1-\delta)\eta q k')$$

- Government imposes a tax τ^D on the bank when it sells seized capital on the secondary market
- The price of debt is pinned down by the break-even condition

$$F(b',k',l',z) = E[\Lambda'(\pi_d\{\mathbb{I}_{n^{e\prime}\geq 0'}b' + \mathbb{I}_{n^{e\prime}<0}\min(b',(1-\tau^D)(1-\delta)\eta q k')\}+$$
$$(1-\pi_d)\{(1-\mathbb{I}_{n'<\underline{n}(z',k')}\mathbb{I}_{n^{e\prime}<0})b' + \mathbb{I}_{n'<\underline{n}(z',k')}\mathbb{I}_{n^{e\prime}<0}\min(b',(1-\tau^D)(1-\delta)\eta q k')\}|z] \quad (11)$$

Equ (11) and Equ (10) constitute a fixed point problem. Debt price and default threshold can be solved simultaneously.

New Entrants

- New entrants replace exit firms and keep the mass of continuing firms equal to one.
- New entrants will be injected with n_0 equity by the representative entrepreneur family
- Draw initial productivity z_0 from the ergodic distribution $\Gamma(z)$

The continuation value for the new entrant is $\tilde{V}(n_0, 0, z_0)$. Let μ be the measure of new entrants:

$$\mu = \pi_d + (1 - \pi_d) \int_0^1 \mathbb{I}[V_i^{exit} > \tilde{V}_i] di \tag{12}$$

Capital Goods Producer

Representative capital goods producer who produces new capital goods using technology

$$\Phi(\frac{I}{K})K = [\frac{\tilde{\delta}^{1/\phi}}{1-1/\phi}(\frac{I}{K})^{1-1/\phi} - \frac{\tilde{\delta}}{\phi-1}]K \tag{13}$$

where $\tilde{\delta}$ is the aggregate depreciation rate in the steady state that takes capital loss due to irreversibility into account

$$\max_I q\Phi(\frac{I}{K})K - I$$

FOC:

$$q = \frac{1}{\Phi'(\frac{I}{K})} = (\frac{I/K}{\tilde{\delta}})^{1/\phi} \tag{14}$$

The functional form ensures

- $q = 1$ in the steady state
- The producer makes zero profit in the S.S.

Households

A fraction $1 - M$ of households, which we refer to as "Ricardian," have unconstrained access to financial markets and own all the firms.

- can trade one-period risk-free bonds
- $$C^R + B^{R\prime} = wL^R - T^R + \int_0^1 d_i di + B^R(1+r) - \mu n_0 + D^K + D^B$$
$$\Lambda' = \beta \frac{U_c(C^{R'}, L^{R'})}{U_c(C^R, L^R)}, 1 = E[\beta \frac{U_c(C^{R'}, L^{R'})}{U_c(C^R, L^R)} R], w = \frac{U_L(C^R, L^R)}{U_C(C^R, L^R)} \tag{15}$$

A fraction M of households, referred to as "H to M" consume their labor income net of taxes (or transfers) each period, because they do not have access to financial markets.

$$C^K = WK_t^K - T^K \tag{16}$$

Two types of agents with GHH utility function $\log(C_t - \Psi \frac{L_t^{1+\theta}}{1+\theta})$

Government

The government uses lump sum tax collecting from households, tax revenue collecting from banks, and new bond issuance to finance government expenditure and old bond payments.

$$G_t + B_{t+1}^G = T_t^R + T_t^K + (1 + R_t)B_t^G + \int_0^1 (\mathbb{I}_{V_{it}=0} \times \tau_D \eta k_{it})\ di$$

Assumption (1)
If the government only collects lump sum tax from the representative entrepreneur family such that $\{T_t^K\}_{t=0}^{\infty} = 0$*, Ricardian equivalence holds. Without a loss of generosity, we assume* $\{B_t^G\}_{t=0}^{\infty} = 0$

In the benchmark case, we assume $G_t = \overline{G}$.

Definition of Recursive Competitive Equilibrium

Now we describe the steady state recursive competitive equilibrium. The state space is denoted by $\mathcal{S} \equiv \mathcal{N} \times \mathcal{K} \times \mathcal{Z}$. Let $\Sigma_{\mathcal{S}}$ be the sigma algebra on $\mathcal{S}$ and $(\mathcal{S}, \Sigma_{\mathcal{S}})$ the corresponding measurable space. Denote the stationary distribution as λ.

A competitive equilibrium is a value function $\{V(s), \tilde{V}(s), V^{exit}(s)\}$; intermediate firm decision rules $\{d(s), k'(s), b'(s), \ell'(s)\}$; prices $\{r, w, q\}$, debt price schedule $F(b', k', \ell', z)$, default threshold $\underline{n}(k, z)$, Aggregates $\{C^R, C^K, L^R, L^K, I, G\}$ and and measures of agents λ, such that:

1. Household choices are determined by (15), (16)
2. Capital good producer optimize (14)
3. Given prices, intermediate firm's decision rules solve the continuing firm's problem 7 while $V(s), \tilde{V}(s), V^{exit}(s)$ are associated value functions following (6), (5).
4. The bank price default risk competitively (11), consistent with default threshold (10)

Market Clearing Conditions

5 Labor market clears

$$(1-M)L^R + ML^K = (1-\pi_d)\int \mathbb{I}_{\tilde{V}>V^{exit}}\ell' d\lambda + \mu \int \ell'(n_0,0,z)d\Gamma(z) \tag{17}$$

6 Capital market clears

$$\Phi(\frac{I}{K})K = \underbrace{(1-\pi_d)\int \mathbb{I}_{\tilde{V}>V^{exit}}[\mathbb{I}_{k'>(1-\delta)k}(k'-(1-\delta)k) - \mathbb{I}_{k'<(1-\delta)k})((1-\delta)k-k')]\ d\lambda}_{\text{investment by incumbent firms}}$$

$$+\underbrace{[\mu_t \int k'(n_0,0,z)d\Gamma(z)]}_{\text{investment by entry firms}} - \underbrace{\pi_d \int (1-\delta)\eta k\ d\lambda_t + (1-\pi_d)\int \mathbb{I}_{\tilde{V}<V^{exit}}(1-\delta)\eta k\ d\lambda}_{\text{selling of used capital by exiting firms}} \tag{18}$$

7 Goods market clears

$$Y = C + I + G \tag{19}$$

8 The distribution of intermediate firms is consistent with the law of motion Γ implied by decision rules

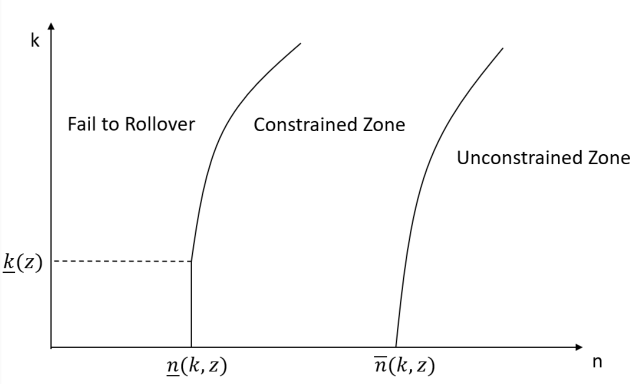
Outline

- Model
- **Model Characterization**
- Empirical Evidence
- Calibration
- Quantitative Results

Decision Threshold for Those Who Consider Continuation

Proposition (1)

In the steady state, consider a firm that does not receive the death shock, has idiosyncratic productivity z, capital k, and has net worth n. There exists a threshold $\underline{n}(k,z)$*, such that the firm defaults when* $n < \underline{n}(k,z)$*.* $\underline{n}(z,k)$ *is weakly increasing in* k*. And there exists* $\underline{k}(z)$ *such that* $\underline{n}(z,k) = \underline{n}(z,0)$ *all for* $k \leq \underline{k}(z)$*.* $\underline{k}(z)$ *is increasing in* z*.* $V(n,k,z) = V^{exit}$ *for* $n < \underline{n}(k,z)$*.*



Definition of Default Threshold
Regression Results

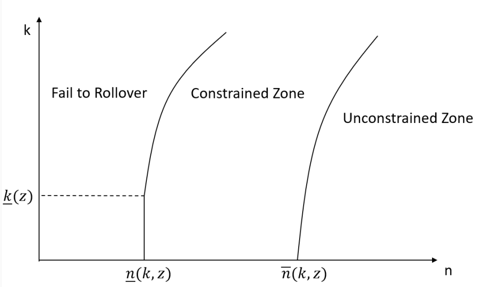
Proposition (2)

Let ϕ be the Lagrange multiplier on (9). There exists a threshold $\overline{n}(k,z)$, such that $\phi = 0$ and the borrowing constraint does not bind onwards when $n > \overline{n}(k,z)$. $\overline{n}(k,z)$ is weakly increasing in k.

$$\tilde{V}(n,k,z) = n - \overline{n}(k,z) + \tilde{V}(\overline{n}(k,z),k,z)$$

Proposition (3)

When $\overline{n}(k,z) > n > \underline{n}(z,k)$, $\phi > 0$ and constrained firms pay zero dividends. The marginal value of liquid wealth is $1 + \phi$.



Decision Rules for Unconstrained Firms

The continuing firm's problem reduces to:

$$\tilde{V}(n,k,z) = n - \overline{n}(k,z) + \tilde{V}(\overline{n}(k,z),k,z)$$

Dividend payment is $d = n - \overline{n}(k,z)$. Then we solve the following problem:

$$\tilde{V}(\overline{n}(k,z),k,z) = \max\{\tilde{V}^{+}(\overline{n}(k,z),k,z), \tilde{V}^{-}(\overline{n}(k,z),k,z)\} \tag{20}$$

Let $k^{+}(k,z)$ be the solution to the upward adjustment problem and $k^{-}(k,z)$ be the solution to the downward adjustment problem. Let us define the marginal Q, as the marginal benefit of investment:

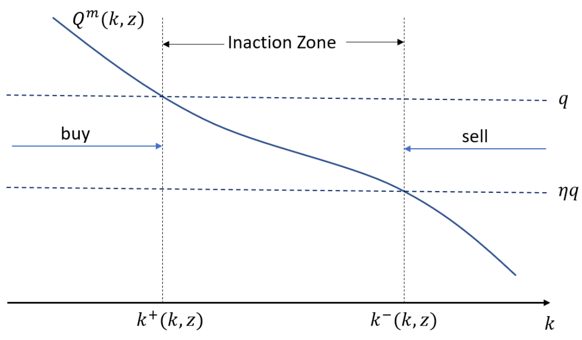
$$\begin{aligned}
Q^{m}(k,z) &\equiv \mathbb{E}[\Lambda' \frac{dV(n',k',z')}{dk'}|z] \\
&= \mathbb{E}[\Lambda'(\frac{\partial V(n',k',z')}{\partial k'} + \frac{\partial V(n',k',z')}{\partial n'}\frac{\partial n'}{\partial k'})|z] \\
&= \mathbb{E}[\Lambda'(\underbrace{\frac{\partial V(n',k',z')}{\partial k'}}_{<0\text{:irreversibility punishment}} + \underbrace{\frac{\partial V(n',k',z')}{\partial n'} \times MRPK'}_{>0\text{:liquid wealth}} + \underbrace{\frac{\partial V(n',k',z')}{\partial n'} \times (1-\delta)q)}_{>0\text{:illiquid wealth}} |z] \\
&= \mathbb{E}[\Lambda'((\frac{\partial V(n',k',z')}{\partial k'} + MRPK') + (1-\delta)q)|z]
\end{aligned}$$

Decision Rules for Unconstrained Firms

Following the first-order conditions and the envelop conditions. We have:

$$Q^m(k^+;k,z) = \mathbb{E}[\Lambda' \frac{dV'}{\partial k'}|k' = k^+] = q, Q^m(k^-;k,z) = \mathbb{E}[\Lambda' \frac{dV'}{\partial k'}|k' = k^-] = \eta q$$

Figure 1: Decision Rules Among Unconstrained Firms



Decisions Rules for Constrained Firms: Debt Price

Among constrained firms in the subset of state space $\mathcal{S}$, investment policy will depend on both (n, k, z) and $d(n, k, z) = 0$. Similarly, we solve the following problem:

$$\tilde{V}(n, k, z) = \max\{\tilde{V}^{+}(n, k, z), \tilde{V}^{-}(n, k, z)\} \tag{21}$$

F.O.C. with respect to b' delivers:

$$\underbrace{\frac{\Lambda'}{\frac{\partial F}{\partial b'}} - 1}_{\text{credit spread}} = \frac{\frac{\partial \tilde{V}(n,k,z)}{\partial n}}{\mathbb{E}[\frac{\partial V(n',k',z')}{\partial n'}]} - 1$$

If the current period borrowing does not involve default risks:

$$1 + \phi(n, k, z) = \frac{\partial \tilde{V}(n, k, z)}{\partial n} = \mathbb{E}[\frac{\partial V(n', k', z')}{\partial n'}] = 1 + \mathbb{E}[\phi(n', k', z')|z]$$

If the current period of borrowing does involve default risks in the next period:

$$1 + \phi(n, k, z) = \frac{\partial \tilde{V}(n, k, z)}{\partial n} > \mathbb{E}[\frac{\partial V(n', k', z')}{\partial n'}] = 1 + \mathbb{E}[\phi(n', k', z')|z]$$

Decisions Rules for Constrained Firms: Investment Wedge

Regression Results

Labor Wedge

$$w = \mathbb{E}[\Lambda'\mathsf{MRPL}'] + \underbrace{\frac{\partial F}{\partial l'} + \mathbb{E}[\Lambda' \ (\overbrace{\frac{\frac{\partial V}{\partial n'}}{\frac{\partial \tilde{V}}{\partial n}} - 1}^{\text{default premium}}) \ \mathsf{MRPL}']}_{\text{Labor Wegde}}$$

Investment Wedge:

$$\underbrace{q(1-\mathbb{E}[\Lambda'(1-\delta)])}_{\text{use cost: upward adjustment}} = \mathbb{E}[\Lambda'\mathsf{MRPK}'] - \underbrace{\mathbb{E}[\Lambda' \ (\overbrace{1 - \frac{\frac{\partial V}{\partial n'}}{\frac{\partial \tilde{V}}{\partial n}}}^{\text{default premium}}) \ (\mathsf{MRPK}' + (1-\delta)q)] + \mathbb{E}[\Lambda' \frac{\frac{\partial V}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}] + \frac{\frac{\partial F}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}}_{\text{Investment Wedge}} \quad (22)$$

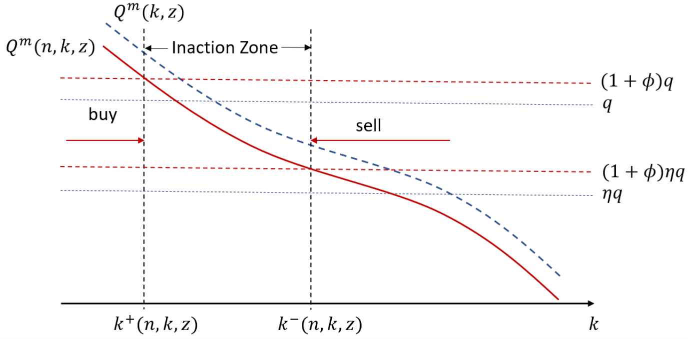
$$\underbrace{\eta q(1-\mathbb{E}[\Lambda'(1-\delta)])}_{\text{use cost: downward adjustment}} = \mathbb{E}[\Lambda'\mathsf{MRPK}'] - \underbrace{\mathbb{E}[\Lambda' \ (\overbrace{1 - \frac{\frac{\partial V}{\partial n'}}{\frac{\partial \tilde{V}}{\partial n}}}^{\text{default premium}}) \ (\mathsf{MRPK}' + (1-\delta)q)] + \mathbb{E}[\Lambda' \frac{\frac{\partial V}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}] + \frac{\frac{\partial F}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}}_{\text{Investment Wedge}} \quad (23)$$

Following the first-order conditions w.r.t k' and the envelop conditions. We have:

$$Q^m(k^+; n, k, z) = q(1+\phi); Q^m(k^-; n, k, z) = \eta q(1+\phi)$$

$$Q^m(n,k,z) = \mathbb{E}[\Lambda'(\underbrace{\frac{\partial V(n',k',z')}{\partial k'}}_{<0\text{:irreversibility punishment}} + \underbrace{\frac{\partial V}{\partial n'} \times \text{MRPK}'}_{>0\text{:liquid wealth}} + \underbrace{\frac{\partial V}{\partial n'} \times (1-\delta)q)}_{>0\text{:illiquid wealth}} |z] + \frac{\partial F}{\partial k'} \tag{24}$$

Figure 2: Decision Rules Among Constrained Firms



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China Manufacturing Data

Annual Survey of Industrial Firms 1998-2013 (no 2010)

- covers all the firms with revenue over 5 million RMB (20 million RMB from 2011)
- 100K to 300K firms a year, 3.2 M in 14 years
- small and medium sized firms who rely on bank credit and face tight financing constraint
- merge and acquisition are not main source of exiting choices

Variables:

- Production function estimation: value added, fixed capital, wage bill
- Fixed capital investment
- Balance sheet: total asset, total debt, interest payment

What Explains Default

Proposition 1

$$E_{it} = \alpha_i + \alpha_{st} + \beta_1 \log n_{it} + \beta_2 \log z_{it} + \beta_3 \log k_{it} + \beta_4 \mathbb{I}\{i_t \leq 0\} \times \log k_{it} + \Gamma' Z_{i,t-1} + e_{it} \quad (25)$$

	(1) Exit	(2) Exit	(3) Exit	(4) Exit
Log(Net Worth)× D(Net Worth>0)	-0.010***	-0.010***	-0.010***	-0.010***
	(0.000)	(0.000)	(0.000)	(0.000)
Log(-Net Worth)× D(Net Worth<0)	0.008***	0.008***	0.020***	0.020***
	(0.000)	(0.000)	(0.000)	(0.000)
Log(TFPR)	-0.011***	-0.011***	-0.016***	-0.016***
	(0.001)	(0.001)	(0.001)	(0.001)
Log(Fixed Capital)	-0.014***	-0.011***	-0.008***	-0.007***
	(0.000)	(0.000)	(0.000)	(0.000)
D(Investment≤0)× Log(Fixed Capital)		**0.016*****		**0.018*****
		(0.000)		(0.000)
Constant	0.309***	0.299***	0.230***	0.213***
	(0.001)	(0.001)	(0.004)	(0.004)
Year × Sector FE	Yes	Yes	Yes	Yes
Firm FE	No	No	Yes	Yes
Observations	1932468	1932468	1779519	1779519
R-Square	0.062	0.064	0.370	0.373

Notes: Results from estimating (25). Fixed capital k_{it} includes assets in equipment, buildings, structures, and other productive capital measured in terms of the book value. The investment is fixed capital investment $i_t = k_{i,t+1} - k_{it} + \delta_{it}$. Net worth is total assets minus total liabilities. TPFR is residuals from a production function estimation. Data source: Chinese Annual Survey of Industrial Firms, 1998-2007. Standard errors are clustered at the firm level in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Correlation Between Financing Constraint and MRPK

$$\log y_{it} = \alpha_i + \alpha_{st} + \beta_1 \log x_{it} + \beta_2 \mathbb{I}\{i_{it} = 0\} \times \log x_{it} + \Gamma' Z_{i,t-1} + e_{it} \quad (26)$$

$x_{it} \in \{\text{leverage ratio, borrowing rate}\}$, y_{it} is ARPK

	(1) Log(ARPK)	(2) Log(ARPK)
Log(Debt/Asset)	0.054***	
	(0.002)	
D(Investment=0)× Log(Debt/Asset)	**-0.074*** **	
	(0.002)	
Log (Borrowing Rate)		0.033***
		(0.001)
D(Investment=0) × Log(Borrowing Rate)		**-0.017*** **
		(0.001)
Constant	0.160***	0.072***
	(0.001)	(0.004)
Year × Sector FE	Yes	Yes
Firm FE	Yes	Yes
Observations	2875653	2140368
R-Square	0.839	0.855

Notes: Results from estimating (26). α_i is a firm i fixed effect, α_{st} is a year-two digit sector fixed effect, $\mathbb{I}\{i_t = 0\}$ is an indicator specifies whether one makes a positive adjustment in fixed capital or not. ARPK y_{it} is defined as value-added over fixed capital to measure MRPK. The tightness of financing constraint is proxied by two variables $x_{it} \in \{\text{leverage ratio, borrowing rate}\}$. The leverage ratio is defined as total liabilities over total assets, and the borrowing rate is defined as total interest payment over total liabilities. Data source: Chinese Annual Survey of Industrial Firms, 1998-2007. Standard errors are clustered at the firm level in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Sector Level Evidence

What is the role of irreversibility in propagating aggregate shocks? Kermani and Ma (2023) computes sector-specific fixed capital recovery rate in the US using bankruptcy documentation.

$$y_{it} = \alpha_i + Q(t, t^2) + \beta_1 D_t + \beta_2 D_t \times \eta_i + \beta_3 D_t \times s_i + e_{it} \tag{27}$$

- α_i is a two-digit sector-level fixed effect
- $y_{it} \in \{\text{St.d.}(\log \text{ARPK}), \log \text{Output}\}$ is two sector-year level
- $Q(t, t^2)$ is a quadratic form that controls the non-linear aggregate trends
- D_t is a dummy variable that equals one in the years of 2008 and 2009
- η_i is the sector-specific capital recovery rate
- s_i is the sector-level export share in 2007

	(1)	(2)	(3)	(4)
Panel A - Dep. Var. St.d. ($\log$ ARPK)				
D(Recession=1)	0.032***	0.076***	0.020***	0.063***
	(0.005)	(0.019)	(0.008)	(0.021)
D(Recession=1)×Recovery Rate		-0.128**		-0.117**
		(0.053)		(0.053)
D(Recession=1)×Export Share			0.050*	0.043
			(0.026)	(0.027)
Constant	1,748.759***	1,748.759***	1,748.759***	1,748.759***
	(341.223)	(339.110)	(340.109)	(338.419)
Observations	420	420	420	420
R-Square	0.759	0.763	0.761	0.764
Panel B - Dep. Var. ($\log$ Output)				
D(Recession=1)	-0.070***	-0.377***	-0.035***	-0.345***
	(0.029)	(0.108)	(0.045)	(0.117)
D(Recession=1)×Recovery Rate		0.884***		0.859**
		(0.299)		(0.302)
D(Recession=1)×Export Share			-0.156	-0.103
			(0.151)	(0.151)
Constant	-1,035.363	-1,035.363	-1,035.363	-1,035.363
	(1,943.420)	(1,924.397)	(1,943.276)	(1,925.731)
Observations	420	420	420	420
R-Square	0.979	0.979	0.979	0.979

Notes: The table reports results from estimating 27. where α_i is a two-digit sector-level fixed effect. $Q(t, t^2)$ is a quadratic form that controls the non-linear common trend. $y_{it} \in \{\text{St.d.}(\log \text{ARPK}), \log \text{Output}\}$ is two sector-year level outcomes. D_t is a dummy variable that equals one in the years of 2008 and 2009. η_i is the sector-specific capital recovery rate taken from Kermani and Ma (2023). s_i is the sector-level export share defined as export value over output in 2007. Standard errors are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Outline

- Model
- Model Characterization
- Empirical Evidence
- **Calibration**
- Quantitative Results

Externally Calibrated Parameters

Table 1: Summary of Externally Calibrated Parameters

Parameters	Description/Sources	Value
Households		
β	Discounting factor	0.99
ν	Inverse of elasticity of labor supply	1
M	Measure of entrepreneur family	0.5
Firms		
α	Capital share	0.30
δ	Depreciation rate	0.025
θ	Elasticity of substitution	5.0
Φ	Elasticity of q to I/K	2
Idiosyncratic Processes		
ρ	From micro estimates	0.75
σ_ν	From micro estimates	0.08
σ_ε	From micro estimates	0.18
Policies		
$\overline{G}$	$\overline{G}/Y = 0.25$	0.42
λ^G	Elasticity of fiscal policy	1
$\overline{\tau^D}$	Collateral parameter in S.S.	0.2
λ^{τ^D}	Elasticity of credit policy	4

Internally Calibrated Parameters

Table 2: Summary of Internally Calibrated Parameters

Parameters	Description	Value
ξ	Operating cost	0.15
π_d	Exogenous exit rate	0.02
n_0	Initial equity	0.18
η	Recovery rate	0.43

Table 3: Calibration Target and Model Fit

Moments	Description	Data	Model
$\mathbb{E}$[default rate]	Annual default rate	0.04	0.04
$\mathbb{E}$[Exit rate]	Annual exit rate	0.12	0.12
$\mathbb{E}[\frac{n(age \leq 3\text{years})}{\overline{n}}]$	Ratio of net worth at age 1-3 to mean	0.49	0.48
$\mathbb{E}$[Investment=0]	Ratio of annual inactive firms	0.21	0.21

Outline

- Model
- Model Characterization
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- **Quantitative Results**

Aggregate Responses to Productivity Shock

The TFP in the steady state is $\overline{A} = 1$, the shock I consider is:

$$\log A_{t+1} = \rho_A \log A_t + (1 - \rho_A) \log \overline{A} + \varepsilon_t^A$$

where the economy starts from the steady state and TFP shock hits at t_0 unexpectedly while $\varepsilon_{t_0}^A = -0.02$ and $\varepsilon_{t \neq t_0}^A = 0$. The persistence of the shock is $\rho_A = 0.7$.

Proposition (4)

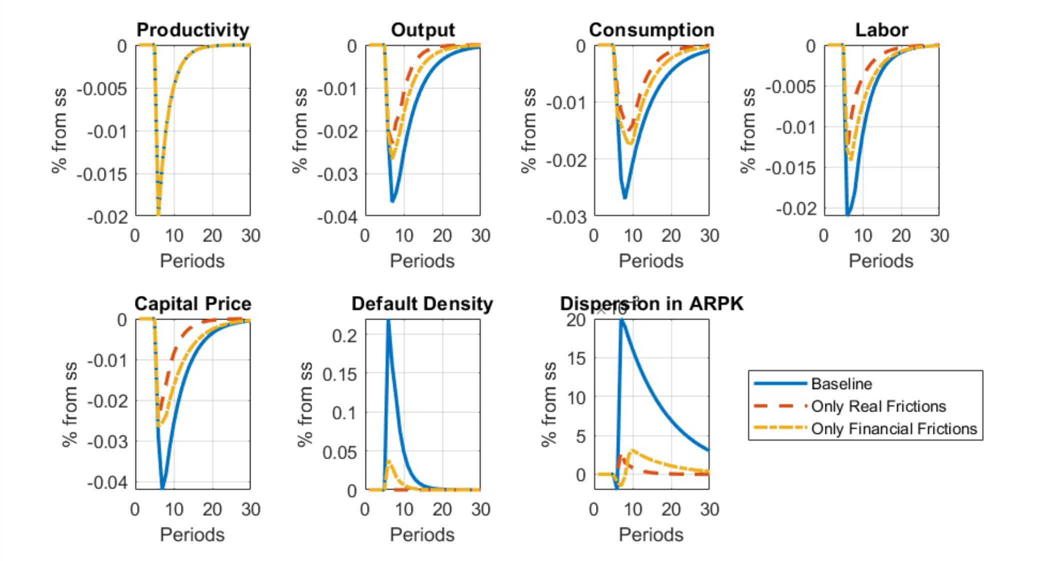
Along the transition dynamics with $\varepsilon_{t_0}^A < 0$, the default threshold is higher than in the steady state

$$\{\underline{n}_t(k, z)\}_{t=t_0}^{\infty} > \underline{n}(k, z)$$

and the threshold specifies the default threshold dependency on k is lower

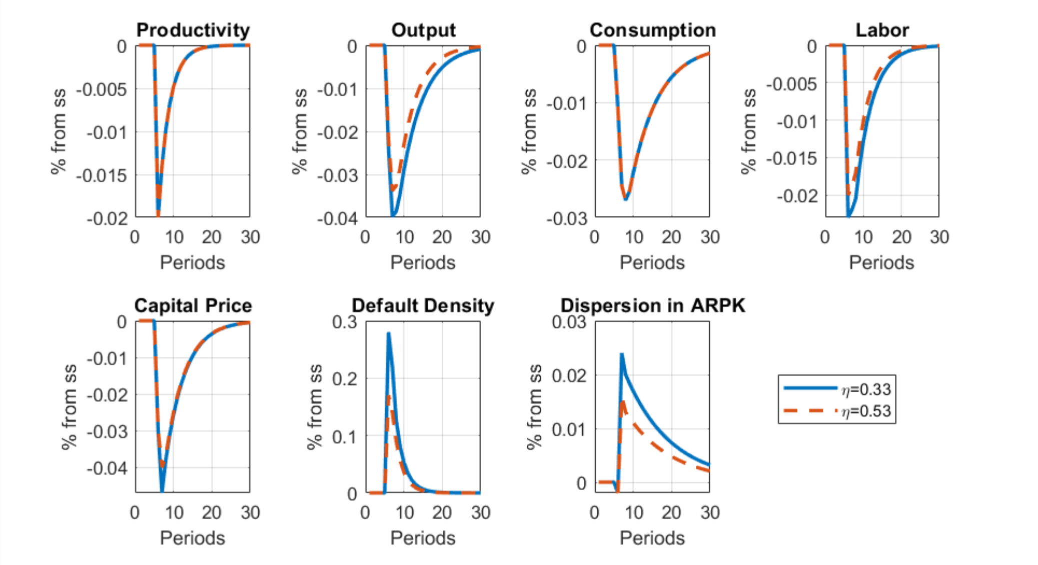
$$\{\underline{k}_t(z)\}_{t=t_0}^{\infty} < \underline{k}(z)$$

- The full model with both financial frictions and irreversibility
- The model without financial frictions
- The model without real frictions, $\eta = 1$



Notes: Aggregate impulse responses to 2 percent negative TFP shock which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6.

IRFs to TFP Shock: Two Sector Model



Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6.

Counter Cyclical Fiscal Policy

What are the inefficiencies in this economy?

- Misallocation, Inefficient Liquidation and Default
- Amplified by irreversibility, hand to mouth households

$$\frac{p_{it}y_{it}}{P_t} = (A_t z_{it} \varepsilon_{it} k_{it}^{\alpha} \ell_{it}^{1-\alpha})^{1-\frac{1}{\theta}} \times Y_t^{\frac{1}{\theta}}$$

Creates two externalities

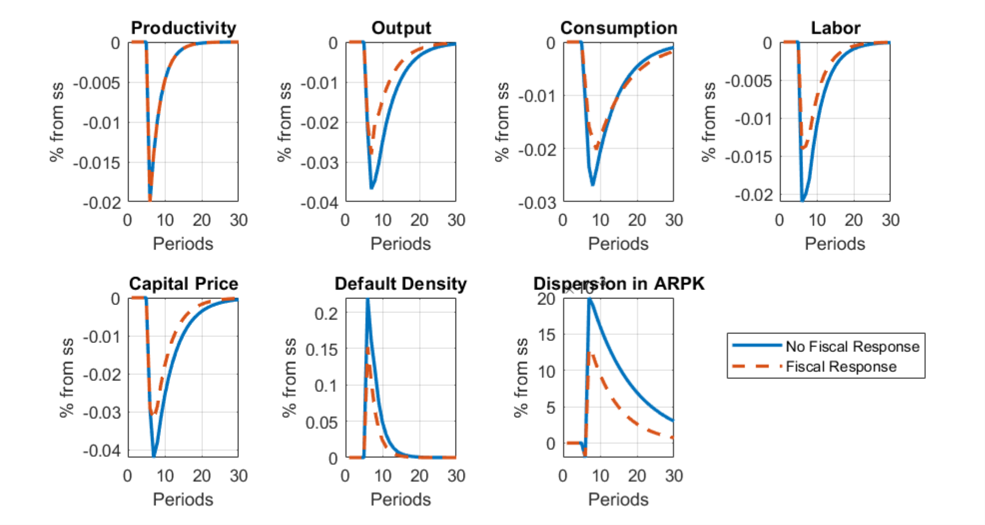
- Aggregate demand externality
- Pecuniary externality

$$\log G_t = \log \overline{G} - \lambda^G (\log Y_{t-1} - \log \overline{Y})$$

Government expenditure

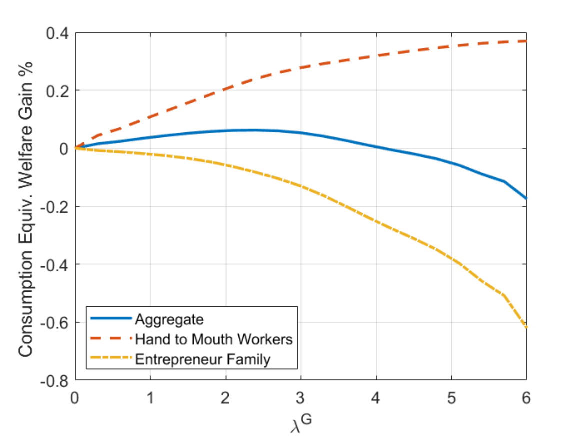
- Release firms from investment inaction zone
- Relax financing constraint endogenously

IRFs to TFP Shock: Fiscal Policy



Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6. The solid lines depict the IRFs in the baseline model with no active fiscal policy, and the dashed lines depict the IRFs in the model with counter-cyclical fiscal policy as described in the text. $\lambda^G = 1$

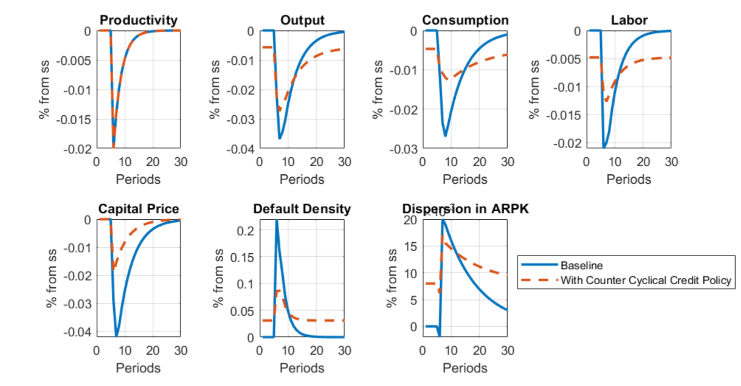
Figure 3: Consumption Equivalent Welfare Gains under Fiscal Policy



Notes: Consumption equivalent welfare gains under various λ^G along the transition path in responses to 2 percent negative TFP shock, which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state. The solid line presents welfare gains for the aggregate economy with equal weight, the dashed line presents welfare gains of the Hand-to-Mouth workers, and the dash-dot line presents welfare gains of the entrepreneur family.

Government can also choose $\tau^D \in [0, 1]$ to control credit supply where the policy rule can be specified as:

$$\tau_t^D = \max\{\overline{\tau^D} + \lambda^{\tau^D}(\log Y_{t-1} - \log \overline{Y}), 0\}$$



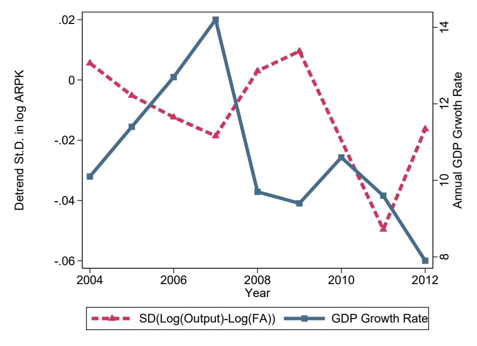
Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6. The solid lines depict the IRFs in the baseline model with no active credit policy, and the dashed lines depict the IRFs in the model with counter-cyclical credit policy as described in the text. $\lambda^{\tau^D} = 4$

Conclusion

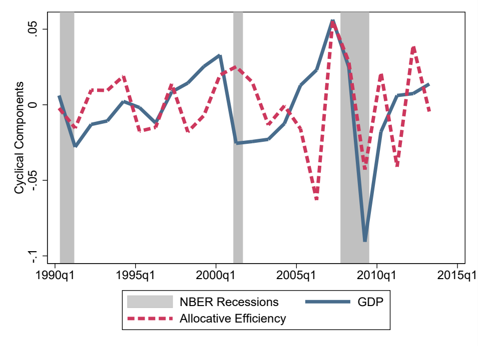
- Build a model features financial frictions and partial irreversibility of fixed capital
- Show that the interaction matters for decision rules at the individual level
- Validate the model by micro data
- With the calibrated model, I find the interaction is quantitatively important to generate amplification in response to first-moment shocks
- Modest counter-cyclical fiscal policy is welfare improving

Dispersion in ARPK is counter cyclical

Figure 4: Counter Cyclical Dispersion in ARPK



China



U.S.

Notes: Panel A: Data Source: St.D. of ARPK is computed from the Annual Survey of Industrial firms 2004-2012 (no 2010) in China within the 4-digit sector at annual frequency.
Panel B: Log deviations from HP trend (smoothing parameter $\lambda = 6.25$) of (i) Allocation efficiency data is from (**?**). (ii) US real GDP, manufacturing sector. Annual frequency. Corr(allocative efficiency, GDP)=0.32