Aggregate Demand and Irreversible Investment[*]

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Abstract

This article studies the propagation of aggregate shocks through the interaction between financial frictions and the partial irreversibility of fixed capital and relevant policies. In response to adverse shocks to the level of revenue productivity, firms find it costly to shrink since fixed capital is partially irreversible. This limits cash flow from liquidating fixed capital and tightens the financing constraint on working capital. Insufficient working capital further lowers marginal product of capital and triggers additional fire sales when the marginal value of liquid wealth is high. Credit spreads and default risks rise given unbalanced and unprofitable production. Misallocation and extra default lead to additional output loss and lower capital prices. In the presence of monopolistic competition, lower aggregate demand(output) enters revenue productivity and creates an endogenous feedback loop. I provide empirical evidence consistent with the model at the firm, sector, and aggregate level. Quantitative results show that 2 percent TFP shock generates a maximum of 3.9 percent drop in output. A counter-cyclical fiscal policy can mitigate the recession by boosting aggregate demand, resulting from a rich interaction between liquidity, asset price, and financial frictions.

1   Introduction

One of the key research questions in macroeconomics is to what extent micro-level frictions and heterogeneity are important to understanding the dynamics of aggregate variables. This article studies the propagation of first-moment aggregate shocks through the interaction between financial frictions and the partial irreversibility of fixed capital and policies to mitigate recessions. Partial irreversibility and financial frictions have been separately studied extensively in the literature [1,2]. Although the financial frictions and irreversibilities have been shown to be matching micro data on investment and financing decisions [3], each has been found to generate relatively little amplification of aggregate shocks to the level of revenue productivity. Specifically, aggregate dynamics are quantitatively similar to those in a standard RBC model with first-moment shocks [4,5]. One requires exogenous financial shocks and uncertainty (second-moment) shocks to make the micro-level frictions matter for aggregate dynamics[6].

In this article, I revisit these irrelevance perspectives and argue that the interaction between partial irreversibility and financial frictions is essential to determine individual firms' investment decisions and generate substantial amplification effects in response to first-moment shocks. To make my argument, I build a quantitative general equilibrium model populated by heterogeneous firms that face idiosyncratic productivity shocks and endogenous default risks under monopolistic competition. Firms use internal funds and external

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[1]Technically speaking, a wedge between the purchasing and selling price of capital summarizes frictions, including asset specificity, indivisibility, and search frictions. Kermani and Ma (2023) shows the wedge is significant. They document that the liquidation value of fixed assets is 35% of the net book value in the average industry in the U.S.. It is also motivated by the observations that gross investment data contains many zeros and is highly asymmetric, see Cooper and Haltiwanger (2006).

[2]It is often modeled as collateral constraints following Kiyotaki and Moore (1997) as debt is not secured with limited commitment. There is a large literature in corporate finance. For example, see Whited (1992) and Bolton et al. (2011).

[3]For example, see Cooper et al. (1999),Cooper and Haltiwanger (2006); Hennessy et al. (2007),Riddick and Whited (2009), Erickson and Whited (2000) and Cao et al. (2019).

[4]The irrelevance result on non-convex adjustment cost is established in a sequential work by Thomas (2002), Khan and Thomas (2008) and House (2014). In short, investment is forward-looking in the presence of non-convex adjustment costs and very sensitive to the change in the price of capital.

[5]Although Kiyotaki and Moore (1997) articulates theoretically how financial frictions affect investment and create capital misallocation, Kocherlakota et al. (2000) shows limited amplification effect of financial frictions in a calibrated model with labor. Khan and Thomas (2013) shows limited misallocation is created in response to aggregate productivity shocks in a quantitative model with collateral constraints.

[6]For example, Bloom (2009) shows ignoring capital adjustment costs would lead to substantial bias in accounting the impacts of uncertainty shocks and Bloom et al. (2007) shows higher uncertainty reduces the responsiveness of investment to demand shocks in the presence of irreversibility.

borrowing to finance both fixed capital and working capital investment. External borrowing is limited since equity issuance is not allowed, and external debt needs to be secured with collateral. Fixed capital is partially irreversible as a constant fraction of capital will be lost when the ownership is transferred.

When calibrated to the Chinese economy, with parameters disciplined by firm-level data in the Chinese manufacturing sector, I find a two percent negative TFP shock leads to a maximum of 3.9 percent drop in output, amplified by a 20 percent increase in default density and around 2 percent increase in the standard deviation of log MRPK. This amplification effect is much smaller in models where I shut down either financial frictions or irreversibility.

To understand the interaction of irreversibility and financial friction and how the two frictions reinforce each other, consider a firm that receives a temporary negative shock to the level of revenue productivity. Suppose there is no friction; it is optimal for the firm to downsize fixed capital and working capital instantly and simultaneously. When we only add partial irreversibility, firms find immediate downsizing costly and might want to hold the fixed capital and enjoy the option value, delivering an investment inaction zone. Meanwhile, firms will also choose a higher working capital as fixed capital and working capital are complements. One key assumption I made is both fixed capital and working capital investment are subject to financing constraints in my model. This is in contrast to most previous work that assumes labor and intermediate inputs are free from financial frictions[7].

With financial frictions on top of irreversibility, suppose firms do not choose to sell fixed capital immediately. Firms would not generate cash flow by liquidating fixed capital, but the funding demand to finance working capital would be higher. As a result, the existence of irreversibility makes financing constraints tighter. Conversely, when the financing constraint on working capital binds, MRPK will be lower since working capital is not financed to the optimal level, and the optimal level of fixed capital holding will be lower, which tightens the "irreversibility constraint" and enlarges the investment inaction zone. Consequently, firms would have too much fixed capital but too little working capital in the presence of the interaction of the two frictions, and misallocation rises. The production would be very unbalanced and unprofitable, while firms are expected to lose net worth quickly. This inefficient input choice is difficult to solve since the fixed capital choice is constrained by irreversibility, and the working capital choice is constrained by financial frictions.

By recognizing that firms are losing net worth, default risks and credit spread pick up.

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[7]Though researchers often get abstraction from financial frictions on working capital, Fazzari and Petersen (1993) do find that working capital competes with fixed capital investment for a limited pool of finance. This argument is also used in the international macro literature for amplification effect see Mendoza (2010).

When a firm is close to the default threshold with a high marginal value of liquid wealth, a fire sale is triggered and one has to sell the fixed capital even though it is very costly. Firms are gambling between deleveraging, selling fixed capital at a shabby price with lower expected default risks but lower option value, and keeping fixed capital stock with a higher expected default risk but compensated by a higher option value conditional on surviving. Formally, endogenous default rises due to limited commitment and incomplete markets as debt is non-state contingent and defaultable. Firms with too weak fundamentals are not able to roll over the debt. The default threshold on the book value of net worth and risk premium can be solved simultaneously in a fixed point problem. I show that this default threshold is weakly increasing in fixed capital conditional on productivity. These results highlight the additional risks of debt overhang when capital is not fully reversible. In response to aggregate adverse shocks to the level of revenue productivity, the default threshold will be higher and rise more for firms with high fixed capital. As a result, a more significant density of firms have to default. Because firms do not produce and are replaced by small and unproductive firms when they default, the effect of default on aggregate output is persistent.

My model features monopolistic competition, while most previous related studies utilize a decreasing return to scale production functions. With monopolistic competition, aggregate demand externality rises à la Blanchard and Kiyotaki (1987). In other words, aggregate demand (aggregate output) enters individual firms' revenue productivity. As a result, output loss caused by misallocation and default will have a spillover effect since aggregate demand is shifted inward. This shift can be viewed as a quasi-negative productivity shock and creates an endogenous feedback loop. Furthermore, the imperfect market setting allows the possibilities of analyzing demand shocks[8] and policies that affect aggregate demand. Additionally, my model assumes a fixed measure of agents are hand-to-mouth workers who only supply labor and have no access to the financial market. When aggregate adverse shocks hit, the labor demand decreases due to exogenous shocks and endogenous feedback through tighter financing constraints, extra default, and lower aggregate demand. A lower labor demand pushes down the wage rate and wage bill for the hand-to-mouth workers. As a result, the consumption demand also contracts, amplified by the supply-side distortion and the existence of hand-to-mouth agents.

Next, I utilize Chinese manufacturing data and provide supportive empirical evidence for my model. First, I test the model-implied decision rules on default and exit. I find that a firm's exit probability is only positively correlated with fixed capital when the firm adjusts its

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[8]See Kiyotaki (2011) for discussion on this.

capital stock non-positively, conditional on net worth and productivity. This is in line with the model that suggests net worth and productivity are not sufficient statistics in deciding the default threshold only if upward fixed capital adjustment is not optimal. Second, I test the model-implied decision rules on fixed and working capital investment. I find that the correlation between the tightness of financial frictions and MRPK is positive for the firms with active investment adjustment, while the correlation is reversed for the firms in the investment inaction zone, a unique feature generated by the interaction between financial frictions and irreversibility implied by the model. Firms equalize the marginal value of liquid wealth and the marginal benefit of investment when making fixed capital investments. When firms are outside the investment inaction zone, a higher marginal value of liquid wealth (tighter financing constraints) is associated with a lower fixed capital choice and higher MRPK. In contrast, when firms are inside the investment inaction zone, a higher marginal value of liquid wealth is associated with a lower working capital choice and lower MRPK. At the sector level, I find that sectors with higher capital recovery rates experience smaller increases in the dispersion of MRPK and a smaller drop in output in recessions, highlighting the role of irreversibility in propagating macroeconomic shocks through misallocation. At the aggregate level, I show that dispersion in MRPK is countercyclical in the manufacturing sector in both China and the US.

Then, I calibrate the model to the Chinese economy. The capital recovery rate, defined as the ratio of liquidation value to book value of fixed capital, governs i) the size of the investment inaction zone and how costly it is to shrink in response to negative shocks, ii) the collateral value of fixed capital, the tightness of financial frictions and how costly of default to the lenders. In the exercises, The capital recovery rate is chosen to match the fraction of inactive firms in the steady state. The default density in the steady state is jointly disciplined by choosing the fixed cost in operation and capital recovery rate to match the empirical non-performing loan rate. With the calibrated model, I fit in ex-ante unexpected and ex-post fully anticipated negative productivity shocks and study the transition dynamics. I find that a 2 percent negative productivity shock generates a maximum of 3.9 percent drop in output, amplified by a 20 percent increase in default density and a 2 percent increase in the standard deviation of MRPK from the steady state. I also find that endogenous feedback through aggregate demand externality accounts for around one-third of the total amplification effect. In contrast, when the irreversibility or financial frictions are shut down, the maximum decline in output is close to 2 percent. In an economy without financial frictions, dispersion in MRPK only increases modestly, and the default channel is shut down. The investment inaction zone

does not move by much because investment is a long-lived object and very elastic to the changes in capital price in the presence of real frictions. In an economy without real friction. Default density only rises modestly, and dispersion in MRPK even goes down. Financial frictions tend to be slightly looser on average in response to aggregate adverse productivity shocks. This is because the decline in asset demand outweighs the decline in collateral value. Capital demand is very elastic to productivity shocks since it becomes a static choice without real frictions, while capital price is forward-looking.

My model also contains novel policy implications. The inefficiency comes from financial frictions and is amplified by the irreversibility. I show that it is beneficial to boost aggregate demand even though my model does not carry any nominal rigidity. In contrast to sticky price, my model features a combination of sticky quantity and financial friction. A higher aggregate demand releases firms from investment inaction zones and relaxes financing constraints endogenously. Better credit conditions improve supply-side efficiency and higher output feedback as additional positive demand shock with self-amplification. Quantitative results show that a counter-cyclical fiscal policy can mitigate recessions by lowering misallocation and default density. Modest welfare gains are achievable even though government expenditure does not enter household utility functions. Along the transition path, the optimal fiscal policy suggests that government expenditure should increase by 2.1 percent in response to a 1 percent drop in GDP. I also show that counter-cyclical credit policy can make the economy more resilient to adverse shocks. However, the tightening financial frictions is associated with significant welfare loss in the steady state.

Related Literature   This article relates to a broad research agenda that studies the implications of micro frictions and heterogeneity in macroeconomics. This article mainly contributes to four strands of literature. First, this paper contributes to our understanding of the propagation and amplification of aggregate shocks through micro-level frictions. Abel and Eberly (1994),Abel and Eberly (1996) and Bertola and Caballero (1994) characterize the optimal decision rules under irreversibility[9]. In response to the first moment shocks, the sole presence of non-convex adjustment cost does not affect aggregate investment by much as shown by Thomas (2002), Khan and Thomas (2008) and House (2014) while Winberry (2021) articulates the elasticity is pro-cyclical. Similarly, in response to first-moment shocks, the sole presence of financial frictions does not deliver a large amplification effect. In con-

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[9]Baley et al. (2022) and Chen et al. (2023) investigate the impact of tax policies in the presence of irreversibility.

trast, second-moment shocks or financial shocks are required to make micro-level frictions relevant.(Gilchrist et al. (2014), Arellano et al. (2019), Khan and Thomas (2013), Bloom et al. (2018)) Existing work that jointly investigates aggregate implications of the two frictions together is limited, with several exceptions. Cui (2022) uses this interaction to rationalize the "capital reallocation puzzle"[10] and Caggese (2007) is the closest to this article, which illustrates this interaction is key to explaining why input inventories and material deliveries are so volatile, procyclical, and asymmetric in industry dynamics. This article shows this interaction is essential in understanding the endogenous amplification in response to first-moment shocks.

Second, this paper contributes to models with financial heterogeneity and endogenous default risk. My model builds on Khan et al. (2014). By applying this framework, Ottonello and Winberry (2020) studies the transmission of monetary policy, and Guntin (2022) studies rollover risks. Compared to their work, I add irreversibility on fixed capital to show that portfolio choice matters for default risks. Additionally, I add monopolistic competition which delivers endogenous amplification through aggregate demand and allows possibilities of analyzing demand shocks.

Third, this paper contributes to our understanding of cyclical properties of misallocation both empirically and theoretically. The effect of misallocation on aggregate output with heterogeneous production units is highlighted by seminal work Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Moll (2014) and Midrigan and Xu (2014) argue that the efficiency loss caused by financial frictions in steady states is small if productivity shocks are persistent. Secular trends of misallocation have been studied in the context of development Buera et al. (2011), Buera and Shin (2013), transition in response to interest rate shocks Gopinath et al. (2017) and heterogeneous markups Baqaee and Farhi (2020). The evidence on the cyclicality of misallocation is relatively thin except for Sandleris and Wright (2014) and Oberfield (2013). I contribute to this literature by characterizing how irreversibility reinforces financial frictions and providing novel empirical evidence on misallocation over business cycles.

Finally, this paper contributes to the growing literature on the distributional effect of macroeconomic policies. Kaplan et al. (2018) studies the transmission of monetary policy with heterogeneous households in the presence of illiquid assets and borrowing constraints. My model shares a similar spirit in the supply side. Financially constrained firms would

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[10]Eisfeldt and Rampini (2006) find capital reallocation is procyclical and the benefits to capital reallocation appear countercyclical. Lanteri (2018) rationalize the same thing without financial frictions but the endogenous price for used capital.

not liquidate fixed capital to finance working capital and behave like "rich hand-to-mouth" firms. To boost aggregate demand in the absence of nominal rigidity, I choose fiscal policy which basically redistributes wealth from agents with low MPC to agents with high MPC (the government). Farhi and Werning (2016) shows the macro-prudential policy can do the same thing by redistributing wealth implicitly when monetary policy is constrained. This distributive externality is also important on the supply side as shown by Lanteri and Rampini (2023). Baqaee et al. (2023) is the closest to this article which studies the transmission of demand shock into supply-side allocative efficiency. The difference is the friction in their work is heterogeneous pass-throughs, while the friction in my paper is the combination of real and financial frictions.

Road Map   Section 2 lays out the model. Section 3 characterizes the model and presents related empirical evidence. Section 4 calibrates and validates the model. Section 5 presents the quantitative results and Section 6 concludes.

2   Model

In this section, I develop a heterogeneous firm model that features financial frictions and partially irreversible fixed capital to study the propagation of first-moment shocks. Following Khan et al. (2014) and Ottonello and Winberry (2020), the core of the economy is a continuum of risk-neutral firms facing uninsurable productivity shocks. They use internal and external resources to finance fixed and working capital investments. External financing is constrained as no equity issuance is allowed, and borrowing is not possible unless the debt is secured with collateral. Fixed capital investment is irreversible, as a constant fraction of capital will be lost when the ownership is transferred. I focus on the steady state equilibrium without aggregate uncertainty and study the transition dynamics in response to an ex-ante unexpected and ex-post fully anticipated

2.1   Environment

Time is discrete and infinite. There are three types of goods in the economy. There exists a continuum of intermediate firms that produce differentiated goods using fixed capital and working capital (labor) under monopolistic competition with a measure of one. There exists a final good producer who aggregates differentiated goods into final goods under perfect

competition. Final goods can be used for consumption, investment, government expenditure, and production of capital goods. At the aggregate level, capital goods cannot be transferred back to final goods. At the individual level, once invested, capital goods become specific to intermediate firms. A constant fraction of fixed capital will be lost for technological reasons when it is sold to second-hand users. After losing a fraction, used capital and new capital are perfect substitutes..

There are two representative households in the economy. There is one representative entrepreneur family with a measure of M that consists of a fixed measure of entrepreneurs. Each family member owns an intermediate firm and operates on their own. However, the entrepreneurs would not eat on their own but pay dividends to the family and make the consumption choice together with full insurance within the family. There is also one representative worker family with a measure of $1 - M$ who only supply labor and consume. The worker's family does not have access to the financial market and they are pure hand-to-mouth ("HTM"). A representative capital goods producer uses aggregate capital and new investment to produce new capital with aggregate adjustment cost. A representative bank issues one-period risk-free bonds and makes one-period non-state contingent defaultable loans at rates that take default risks into account. The representative entrepreneur family owns the capital goods producer and the bank.

2.2 Technology

Intermediate firm i utilizes fixed capital k_{it} and working capital l_{it} to produce differentiated intermediate goods with a constant return to scale technology at time t. However, fixed capital k_{it} and working capital l_{it} are fully financed and chosen one period before at $t - 1$. In our model setting, labor input is the working capital. A_t is the aggregate total factor productivity. The persistent component of idiosyncratic productivity z_{it} follows log AR(1) process and the transitory component of idiosyncratic productivity ε_{it} is drawn from a log-normal distribution i.i.d..

$$y_{it} = A_t z_{it} \varepsilon_{it} k_{it}^{\alpha} l_{it}^{1-\alpha}$$

$$\log z_{i,t} = \rho \log z_{i,t-1} + \nu_{it}, \quad \nu_{it} \sim N(0, \sigma_{\nu}^2) \tag{1}$$

$$\log \varepsilon_{it} \sim N(0, \sigma_{\varepsilon}^2) \tag{2}$$

The quantity of aggregate output, which can be used as either a consumption good or investment good, is a function of many differentiated goods as:

$$Y_t = (\int_0^1 (y_{it})^{\frac{\theta-1}{\theta}} di)^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between differentiated goods and $\theta > 1$. Let p_{it} be the price of intermediate good, and we normalize the price index that corresponds to the above aggregate output to be one:

$$P_t = (\int_0^1 p_{it}^{1-\theta} di)^{\frac{1}{1-\theta}} = 1$$

Under monopolistic competition, intermediary firms face a downward-sloping demand curve and make decisions taking aggregate output as given:

$$\frac{y_{it}}{Y_t} = (\frac{p_{it}}{P_t})^{-\theta}$$

We assume that the model does not carry any nominal rigidity, and there is no inventory. As a result, as inputs are predetermined, the price for differentiated goods is implied by the demand curve directly. Then the revenue for firm i in terms of final good can be written as:

$$\frac{p_{it} y_{it}}{P_t} = (A_t z_{it} \varepsilon_{it} k_{it}^\alpha l_{it}^{1-\alpha})^{1-\frac{1}{\theta}} \times Y_t^{\frac{1}{\theta}} \tag{3}$$

Aggregate demand enters into an individual's revenue production function. As a result, supply-side distortions and demand shocks can shift the aggregate demand Y_t and will affect the real revenue of firms, even though there is no price stickiness.
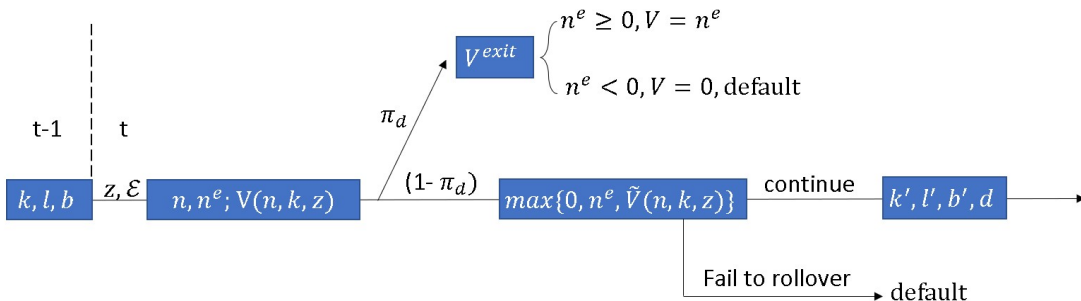
2.3 Intermediary Firms' Problem

Firms are risk-neutral, and the firm i's objective is to maximize the expected discounted value of dividends:

$$V_{i0} = \mathbb{E}_0 \sum_{t \geq 0} \Lambda_t d_{it} \tag{4}$$

where Λ_t is the stochastic discounting factor of the representative entrepreneur family and d_{it} is the dividends paid to the entrepreneur family.

Figure 1: Timeline of Intermediary Firms' Problem

Notes: This figure illustrates the timeline of a typical intermediary firm.

Timeline At the beginning of the period t, intermediary firm i inherits predetermined fixed capital k_{it}, working capital l_{it}, defaultable non-state contingent loan with debt obligation b_{it} from the end of last period. Aggregate and individual level of productivity is realized. From now on, I drop the subscript and use state variables to describe the problem in the steady state. Notice that we have three state variables. persistent component of productivity z enters into the value function because it is persistent and evolves exogenously. Because of the financial frictions, net worth positions matter for the cost of external funding. We define the book value of the net worth as:

$$n = (Az\varepsilon k^{\alpha}l^{1-\alpha})^{1-\frac{1}{\theta}}Y^{\frac{1}{\theta}} + (1-\delta)qk - b \tag{5}$$

where δ is the depreciation rate of fixed capital, and q is the price of fixed capital. Since fixed capital is irreversible, portfolio choices also matter for the firms' choices, and k enters the value function as an endogenous variable.

Then firms receive exogenous death shock with the probability of π_d. The death shock ensures that not all firms save out of financing constraints since they can always delay dividend payments. For firms who receive the exogenous death shock, they choose between exiting after production, paying back debt and dividends, or defaulting on debt straightly. We assume only the one who starts the project can finish the project. A final touch is required to finish production and that will cause some disutility to the firm owners. We also assume there is no time for renegotiation and debt restructuring. As a result, there will be no production under default, leading to deadweight loss. Notice that fixed capital and hired labor will rest for one period.[11] Let $\eta \leq 1$ be the recovery rate when the used capital is sold

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[11]Ottonello (2017) studies capital unemployment with search frictions. Here I assume that fixed capital

in the secondary market by firms or the bank. Suppose the firm finishes production, repays debt, and sells the depreciated used capital in the capital market. The liquidation value is:

$$n^e(n, k, z) = n - (1 - \eta)q(1 - \delta)k \tag{6}$$

The wedge between the book value of the net worth and the liquidation value comes from the partial irreversibility. And there could be a significant fraction of firms that have a positive book value of the net worth but a negative liquidation value.

For firms with non-negative liquidation value $n^e \geq 0$, firms will finish the production and pay n^e as dividends to the family. If the random draws are not so good and firms' fundamentals are also weak such that $n^e < 0$, then firms have no incentive to finish the production and will default on debt. When default happens, fixed capital is seized by creditors as collateral. Firms will exit the economy permanently and pay zero dividends to equity holders. The default value is 0. The exit value can be expressed as:

$$V^{exit}(n, k, z) = \max\{n^e, 0\} = \max\{n - (1 - \delta)(1 - \eta)qk, 0\} \tag{7}$$

With prob $1 - \pi_d$, firms can decide on continuing, exiting endogenously, or defaulting on debt on their own will. Let $V(n, k, z)$ be the value of the firm at the beginning of each period, before receiving the exogenous death shock but after the productivity shocks are realized.

Let $V^{cont}(n, k, z)$ be the continuation value in the middle of the period, and we can connect $V(n, k, z)$ and $V^{cont}(n, k, z)$ as:

$$V(n, k, z) = \pi_d V^{exit}(n, k, z) + (1 - \pi_d) \max\{V^{exit}(n, k, z), V^{cont}(n, k, z)\} \tag{8}$$

Continuation Firms's Problem Let us describe the firm problem conditional on continuing. Firms will choose current period dividends d, next period capital qk', and working capital wl' by raising fund $F(b', k', l', z)$ from the bank where b' is the non-state contingent debt obligation in the next period. Equation 8 and 9 characterize the dynamic programming problem.

Financial frictions take two forms. First, equity issuance is not possible [12], and firms

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will not be used in the production in the period associated with default event. The fixed capital will be sold to a new firm at the end of the period.

[12] This assumption can be relaxed to a costly equity issuance. The Lagrangian multiplier on non-negative dividends payment constraint is equivalent to the extra marginal cost of equity issuance. For example, see

are subject to a non-negative dividend payment constraint 11. This assumption is standard in the literature, and indirect financing through banks dominates in the Chinese economy, which I calibrate to. Secondly, external borrowing $F(b', k', l', z)$ is bounded because firms don't have the full commitment to repay the debt. When default happens, the bank can only seize all the installed capital as collateral with liquidation value $(1 - \delta)\eta k$ but not the potential output even if the materials are ready. $F(b', k', l', z)$ specifies how much one can borrow in this period when one is scheduled to repay b' in the next period conditional on making input choices k', l' and current productivity z. The implied average borrowing rate is $Q(b', k', l', z) = b'/F(b', k', l', z)$. This object is endogenous, and I will discuss the determination of the borrowing rates in detail in the next sections. These lead to a financial constraint with respect to external resources.

Firms are subject to a flow of cash constraint 10. Firms use internal funds n and external borrowing $F(b', k', \ell', z)$ to finance operation fixed cost ξ, investments in fixed capital, working capital $w\ell'$ and dividend payments d. Since fixed capital is irreversible when firms make new investments, the purchasing price of capital is q, and the selling price is ηq when firms make disinvestments.

$$V^{cont}(n, k, z) = \max_{d,k',b',l'} d + \mathbb{E}[\Lambda V(n', k', z')|z] \tag{9}$$

Subject to:

$$d = n + F(b', k', l', z) - \xi - wl' - \mathbb{I}_{k'>(1-\delta)k} \cdot qk' - \mathbb{I}_{k'<(1-\delta)k} \cdot q((1-\eta)(1-\delta)k + \eta k') \tag{10}$$

$$d \geq 0 \tag{11}$$

Default Threshold For Continuing firms Not all continuation firms' problems yield a solution since some of them can't satisfy non-negative dividend payment constraints no matter what choices are made.[13] We can define default threshold $\underline{n}(z, k)$ as the minimum starting net worth such that the maximum of dividends payment one can make is zero.

$$\max_{b',k',l'}\{\underline{n}(z, k) + F(b', k', l', z) - wl' - \xi - \mathbb{I}_{k'>(1-\delta)k} \cdot qk' - \mathbb{I}_{k'<(1-\delta)k} \cdot q((1-\eta)(1-\delta)k + \eta k')\} = 0$$

Lanteri and Rampini (2023) and Guntin (2022).

[13] We let $\tilde{V}(n, k, z) = -\infty$ for firms with $n < \underline{n}(z, k)$.

Rearrange:

$$\underline{n}(z,k) \equiv - \max_{b',k',l'} \left\{ F(b',k',l',z) - wl' - \xi - \mathbb{I}_{k'>(1-\delta)k} \cdot qk' - \mathbb{I}_{k'<(1-\delta)k} \cdot q((1-\eta)(1-\delta)k + \eta k') \right\} \tag{12}$$

Notice that due to the existence of irreversibility constraint, the default threshold depends not only on net worth but also on capital. When $\eta = 1$, $\underline{n}(z,k)$ will be only dependent on productivity and collapse to $\underline{n}(z)$. We will characterize the properties of this default threshold in the next section.

2.4 Bank and Debt Price

The bank is owned by the representative entrepreneur family. It raises funds by issuing one-period risk-free bonds and making one-period non-state contingent deductible loans. The bank takes default risk into consideration when providing financial contracts. When the firms default, the creditors can seize the firms' fixed capital fully. In the case of default, the payment to the creditor will be $\min(b', (1-\delta)\eta qk')$. We assume the banking sector is competitive and not subject to financial frictions. Therefore, it is expected to earn zero profit and risk-neutral. The price of debt is pinned down by the break-even condition:

$$F(b',k',l',b') = \left(\Lambda \left\{ \begin{array}{l} b' \cdot [\pi_d \mathbb{I}_{n^{e'}>0} + (1-\pi_d)(1 - \mathbb{I}_{n'<\underline{n}'}\mathbb{I}_{n^{e'}<0})] + \\ \min(b', \eta q(1-\delta)k) \cdot [\pi_d \mathbb{I}_{n^{e'}<0} + (1-\pi_d)\mathbb{I}_{n'<\underline{n}'}\mathbb{I}_{n^{e'}<0}] \end{array} \right\} \right) \tag{13}$$

LHS specifies how much the bank pays at date t, and LHS gives how much the bank is expected to get paid at date t+1, taking the representative entrepreneur's stochastic discounting factor into account. In the first row of equation 12, banks get paid back entirely when the liquidation value is larger zero $\mathbb{I}_{n^{e'}>0}$ conditional on receiving the death shock, or when the following two does not hold at the same time conditional on not receiving the death shock: (i) liquidation value is negative (ii) net worth is below the default threshold.

In the second row of equation 12, firms would default if the liquidation value is negative conditional on receiving the death shock; or when both the following two holds: (i) liquidation value is negative (ii) the net worth is lower than the default threshold conditional on not receiving the death shock.

Equation 13 and equation 12 constitute a fixed point problem. Debt price and default threshold can be solved simultaneously.

2.5 New entry

In each period, a measure of the same mass firms will replace exit firms and keep the mass of continuing firms equal to one. New entrants will be injected with n_0 equity by the representative household and draw initial productivity z_0 from the ergodic distribution $\Gamma(z)$. Then the firm value for the new entrant is $V(n_0, 0, z_0)$. Note that the initial draw of productivity is only useful to form the expectation. Let μ be the measure of new entrants:

$$\mu_t = \pi_d + (1 - \pi_d) \int_0^1 \mathbb{I}[V_{it}^{exit} > V_{it}^{cont}]di \tag{14}$$

2.6 Capital Goods Producer

There is a representative capital goods producer who produces new capital goods using technology

$$\Phi(\frac{I}{K})K = \Big(\frac{\tilde{\delta}^{1/\phi}}{1 - 1/\phi}(\frac{I_t}{K_t})^{1-1/\phi} - \frac{\tilde{\delta}}{\phi - 1}\Big)K \tag{15}$$

where I is the amount of final goods used to produce capital, K is the aggregate capital stock in the beginning of the period, $\tilde{\delta}$ is the aggregate depreciation rate in the steady state that takes capital loss due to irreversibility into account, and $\Phi()$ is the aggregate capital adjustment cost function. The capital goods producer chooses I to maximize the profit, where q is the price of capital goods.

$$\max_I q\Phi(\frac{I}{K})K - I$$

FOC:

$$q = \frac{1}{\Phi'(\frac{I}{K})} = (\frac{I/K}{\tilde{\delta}})^{1/\phi} \tag{16}$$

In the steady state, $q = 1$ and the price of capital moves with investment in the same direction to capital ratio follows the financial accelerator effect in Bernanke et al. (1999). Furthermore, this function form implies that the capital goods producer makes zero profit in the steady state. The capital goods producer might make net gains or losses D^B which will be transferred back to the representative entrepreneur family lump sum in the transition dynamics.

2.7 Households

There are two representative agents on the demand side with the same GHH preferences over consumption C and Labor supply:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t Log(C_t^j - \Psi \frac{L_t^{j^{1+\nu}}}{1+\nu}), \quad j \in R, K \tag{17}$$

where ψ governs the disutility of labor and ν measures inverse of elasticity of labor supply.

Representative Entrepreneur Family   There exists a representative entrepreneur family with a measure of $1 - M$, and we denote variables she chooses with superscript R. The representative entrepreneur family owns all the intermediate firms, banks, and the representative capital goods producer. The family has access to financial markets and trade one-period risk-free bonds. The flow of cash constraint is described in equation 18. The family uses pre-paid labor income $w_t L_{t+1}^R$, dividend payment $\int_0^1 d_{it} di$, saving in bond $B_t^R (1 + r_t)$, lump sum transfer from the bank and the capital goods producer D_t^B and D_t^K to finance consumption C_t^R, net tax payment to the government T_t^R and bond purchase B_{t+1}^R.

$$C_t^R + B_{t+1}^R = w_t L_{t+1}^R - T_t^R + \int_0^1 d_{it} di + B_t^R (1 + r_t) - \mu_t n_0 + D_t^K + D_t^B \tag{18}$$

The stochastic discounting factor is defined as:

$$\Lambda_t = \beta \frac{U_c(C_{t+1}^R, L_{t+1}^R)}{U_c(C_t^R, L_t^R)} \tag{19}$$

And F.O.C. implies:

$$1 = \mathbb{E}_t[\beta \frac{U_c(C_{t+1}^R, L_{t+1}^R)}{U_c(C_t^R, L_t^R)} R_{t+1}] \tag{20}$$

$$w_t = \mathbb{E}[\frac{\beta U_L(C_{t+1}^R, L_{t+1}^R)}{U_C(C_t^R, L_t^R)}] \tag{21}$$

Representative Worker   There exists a representative worker with a measure of M, and we denote variables she chooses with superscript K. we assume that they do not have access to financial markets. Henceforth referred to as "Hand to Mouth," who just consume their labor income net of taxes (or transfers) each period.

$$C_t^K = w_t L_{t+1}^K - T_t^K \tag{22}$$

2.8   Government

The government uses lump sum tax collecting from households, tax revenue collecting from banks, and new bond issuance to finance government expenditure and old bond payments.

$$G_t + B_{t+1}^G = T_t^R + T_t^K + (1 + R_t) B_t^G$$

Assumption 1 *If the government only collects lump sum tax from the representative entrepreneur family such that $\{T_t^K\}_{t=0}^\infty = 0$, Ricardian equivalence holds. Without a loss of generosity, we assume $\{B_t^G\}_{t=0}^\infty = 0$*

In the benchmark case, we assume $G_t = \overline{G}$.

2.9   Definition of Recursive Competitive Equilibrium

Now we describe the steady state recursive competitive equilibrium. The state space is denoted by $\mathcal{S} \equiv \mathcal{N} \times \mathcal{K} \times \mathcal{Z}$. Let $\Sigma_\mathcal{S}$ be the sigma algebra on $\mathcal{S}$ and $(\mathcal{S}, \Sigma_\mathcal{S})$ the corresponding measurable space. Denote the stationary distribution as λ.

A competitive equilibrium is a value function $\{V(s), V^{cont}(s), V^{exit}(s)\}$; intermediate firm decision rules $\{d(s), k'(s), b'(s), l'(s)\}$; prices $\{r, w, q\}$, debt price schedule $F(b', k', \ell', z)$, default threshold $\underline{n}(k, z)$, Aggregates $\{C^R, C^K, L^R, L^K, I, G\}$ and and measures of agents λ, such that:

1. Household choices are determined by (19), (20), (21)

2. Capital good producer optimize (16)

3. Given prices, intermediate firm's decision rules solve the continuing firm's problem 9 while $V(s), \tilde{V}(s), V^{exit}(s)$ are associated value functions following (8), (7).

4. The bank price default risk competitively (13), consistent with default threshold (12)

5. Labor market clears

$$(1 - M)L^{R'} + ML^{K'} = (1 - \pi_d) \int \mathbb{I}_{V^{cont} > V^{exit}} l' d\lambda + \mu \int l'(n_0, 0, z) d\Gamma(z) \tag{23}$$

6. Capital market clears

$$\Phi(\frac{I}{K})K = \underbrace{(1-\pi_d)\int \mathbb{I}_{V^{cont}>V^{exit}}[\mathbb{I}_{k'>(1-\delta)k}(k'-(1-\delta)k) - \mathbb{I}_{k'<(1-\delta)k}\eta((1-\delta)k-k')]\ d\lambda}_{\text{investment by incumbent firms}}$$

$$\underbrace{+[\mu_t \int k'(n_0,0,z)d\Gamma(z)]}_{\text{investment by entry firms}} - \underbrace{[\pi_d \int (1-\delta)\eta k\ d\lambda_t + (1-\pi_d)\int \mathbb{I}_{V^{cont}<V^{exit}}(1-\delta)\eta k\ d\lambda]}_{\text{selling of used capital by exiting firms}}$$

(24)

7. Goods market clears

$$Y = C + I + G \tag{25}$$

8. The distribution of intermediate firms is consistent with the law of motion Γ implied by decision rules and constitutes a fixed point

3 Model Characterization and Empirical Evidence

In this section, we focus on steady-state and characterize the decision rules in different scenarios. We first split the state space into three zones following 1 and then characterize decision rules in each case. At the same time, we utilize Chinese manufacturing data to examine the model-implied empirical predictions.

3.1 Default, Constrained and Unconstrained Zones

Considering the continuation problem for the intermediate firms in the state space $\mathcal{S}$, not all of the firms are able to rollover the debt and satisfy non-negative dividend payment constraints.

Proposition 1 *In the steady state, consider a firm that does not receive the death shock, has idiosyncratic productivity z, capital k, and has net worth n. There exists a threshold $\underline{n}(k,z)$, such that the firm defaults when $n < \underline{n}(k,z)$. $\underline{n}(z,k)$ is weakly increasing in k. And there exists $\underline{k}(z)$ such that $\underline{n}(z,k) = \underline{n}(z,0)$ all for $k \leq \underline{k}(z)$. $\underline{k}(z)$ is increasing in z. $V(n,k,z) = V^{exit}$ for $n < \underline{n}(k,z)$.*

Proof: See Appendix C.

To understand this proposition, it is convenient to see (12). If there is no irreversibility where $\eta = 1$, the default threshold collapses to be just a function of productivity z[14]. With irreversibility where $\eta < 1$, the default threshold if and only if be a function of both productivity z and fixed capital k when capital downward adjustment solves the maximization problem in (12). $k(z)$ specifies the threshold when downward adjustment maximizes the payment of the dividends. The key message is that the default threshold will be independent of k when upward adjustment is the optimal choice. But that will be dependent on k when downward adjustment is the optimal choice.

Proposition 2 *Let ϕ be the Lagrangian multiplier on the non-negative dividend constraint 11. There exists a threshold $\overline{n}(k, z)$, such that the non-negative dividend constraint is not binding, which implies that the borrowing constraint does not bind onward when $n > \overline{n}(k, z)$. $\overline{n}(k, z)$ is weakly increasing in k.*

$$V^{cont}(n, k, z) = n - \overline{n}(k, z) + V^{cont}(\overline{n}(k, z), k, z)$$

Proof: See Appendix C.

Firms who are able to roll over the debt and continue may face financing frictions in making investments. Firms can borrow at a risk-free rate when the current borrowing is associated with no default risk in the next period. However, that does not imply the firm will never face any default risks in the future. Therefore, If a firm places non-zero probability weight on encountering a state in which default risks are non-zero, we identify it as constrained; otherwise, it is unconstrained. To be clear, a constrained firm need not face a debt price with a risk premium in the current period; the definition includes firms perceiving the risks of facing a default probability in the future.

Since intermediate firms are risk-neutral, it is always optimal for firms to retain profit and delay dividend payments, except if one has saved out of financing constraints. Let ϕ_t be the Lagrange multipliers on non-negative dividend payment constraint in a sequential problem. It can be shown that ϕ_t is supermartingale.[15] $\phi = 0$ corresponds to the case where firms save out of financing constraints, and $1 + \phi$ summarizes the marginal value of liquid wealth.

Proposition 2 illustrates the subset of state space where firms save out of financing constraints. For firms that save out of financing constraints, they are indifferent between retain-

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[14]This is the case in Ottonello and Winberry (2020)

[15]See Appendix C for the proof.

ing the profit within the firm or paying back to the firm owners as long as they maintain a minimum net worth position $\overline{n}(k, z)$. As a result, the decision rules of unconstrained firms will be collapsed into a two-dimensional object.

Proposition 3 *When $\overline{n}(k, z) > n > \underline{n}(z, k)$, $\phi > 0$ and constrained firms pay zero dividends. The marginal value of liquid wealth is $1 + \phi$.*

Proof: See Appendix C.

For firms with a net worth between the two thresholds, financial constraints bind, and they would choose not to invest to the optimal level. The tightness of financial frictions can be proxied by the marginal value of liquid wealth. Figure 2 illustrates a plane in the state space $\mathcal{N} \times \mathcal{K}$ can be divided into three regions, following the previous propositions.

Figure 2: Default and Unconstrained Threshold
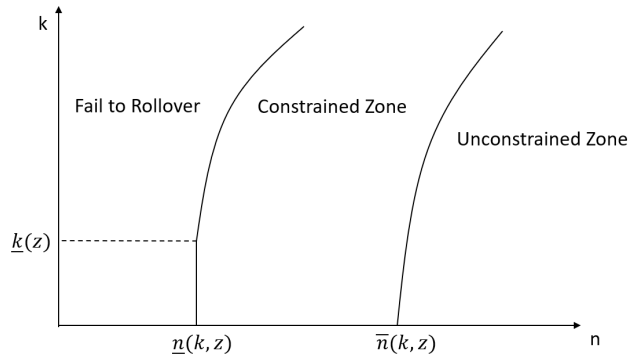
Notes: This figure illustrates the state space $\mathcal{N} \times \mathcal{K}$ conditional on z and the thresholds for default, constrained, and unconstrained zones.

3.2 Decision Rules

Unconstrained Firms For unconstrained firms in the subset of state space $\mathcal{S}$, the problem reduces to a classical investment problem with non-convex adjustment costs. The continuing firm's problem reduces to:

$$V^{cont}(n, k, z) = n - \overline{n}(k, z) + V^{cont}(\overline{n}(k, z), k, z)$$

Dividend payment is undetermined since the marginal value of liquid wealth is identical between the unconstrained intermediate firm and the representative entrepreneur family. As

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a result, $d \in [0, n - \bar{n}(k, z)]$. To consider the optimal investment policy, let us define the marginal $Q^m(k'; z)$ as the present value by having one additional unit of fixed capital [16].

$$Q^m(k'; z) \equiv \mathbb{E}[\Lambda' \frac{dV(n', k', z')}{dk'} | z]$$

$$= \mathbb{E}[\Lambda'(\underbrace{\frac{\partial V(n', k', z')}{\partial k'}}_{<0:\text{irreversibility punishment}} + \underbrace{\frac{\partial V(n', k', z')}{\partial n'} \times \text{MRPK}'}_{>0:\text{liquid wealth}} + \underbrace{\frac{\partial V(n', k', z')}{\partial n'} \times (1 - \delta)q)}_{>0:\text{illiquid wealth}} | z]$$

$$= \mathbb{E}[\Lambda'((\frac{\partial V(n', k', z')}{\partial k'} + \text{MRPK}') + (1 - \delta)q) | z]$$

With one additional unit of capital, firms will generate more liquid wealth by the marginal revenue product of capital at date t+1 by $\text{MRPK}' = \frac{\partial}{\partial k'}[(A'z'\varepsilon'k'^\alpha l'^{1-\alpha})^{1-\frac{1}{\theta}} Y'^{\frac{1}{\theta}}]$. Firms will also retain fixed capital by $(1 - \delta)q$. $\frac{\partial V(n', k', z')}{\partial k'}$ is the marginal effect of extra capital, controlling the book value of net worth at date $t + 1$. Because having a larger capital for a given net worth makes it more difficult for the firm to repay debt and adjust production, this compensated term that summarizes the cost of irreversibility is negative. $\frac{\partial V(n', k', z')}{\partial k'} = 0$ when the fixed capital is fully reversible. The last equality comes from the property that the marginal value of liquid wealth is equal to one when the firms are financially unconstrained.

Following the first-order conditions and the envelop conditions. We have:

$$Q^m(k^+; z) = \mathbb{E}[\Lambda' \frac{dV'}{dk'} | k' = k^+] = q, \quad Q^m(k^-; z) = \mathbb{E}[\Lambda' \frac{dV'}{dk'} | k' = k^-] = \eta q$$

Figure 3 displays the investment policy. Firms will

$$
\begin{cases}
\text{purchase capital to } k' = k^+(z), \text{ if } (1 - \delta)k < k^+(z) \\
\text{sell capital to } k' = k^-(z), \text{ if } (1 - \delta)k > k^-(z) \\
\text{maintain capital at } k' = (1 - \delta)k, \text{ if } k^+(z) < (1 - \delta)k < k^-(z)
\end{cases}
$$

Constrained Firms   Among constrained firms in the subset of state space $\mathcal{S}$, investment policy will depend on both (n, k, z) and $d(n, k, z) = 0$.

F.O.C. with respect to b' delivers the optimal borrowing:

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[16]Notice that we define marginal Q slightly differently from the classical way in two aspects: (i) Our marginal Q does not include current period benefit because net worth is defined after receiving the revenue payment. (ii) We do not divide the replacement cost of fixed capital because we want to display in diagrams how the price of capital affects investment decisions.

Figure 3: Decision Rules Among Unconstrained Firms
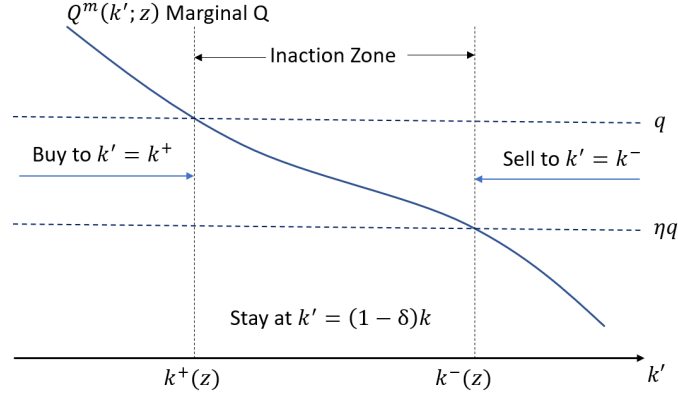
$$\frac{\partial F(b', k', l', z)}{\partial b'} \cdot \frac{\partial V^{cont}(n, k, z)}{\partial n} = \mathbb{E}[\Lambda' \frac{\partial V(n', k', z')}{\partial n'}|z] \tag{26}$$

The left hand side (LHS) is the marginal benefit of raising funds at date t by issuing extra debt to pay at date t+1: The first term is the extra fund raised, and the second term is the marginal value of date-t liquidity. If the non-negativity constraint of dividend (11) is binding at date t so that Lagrangian multiplier ϕ is positive, the marginal value of date-t liquidity exceeds the consumption value:

$$\frac{\partial V^{cont}(n, k, z)}{\partial n} = 1 + \phi > 1$$

The right hand side (RHS) of (26) is the marginal cost of issuing extra debt, as date t+1 book value of the net worth decreases by unity by extra debt issue. We can rewrite the first order condition as:

$$\underbrace{\frac{\partial F(b', k', l', z)}{\partial b'} - \mathbb{E}[\Lambda']}_{<0: \text{ price wedge in risky bond over safe bond}} = \mathbb{E}\left[\Lambda' \underbrace{\left(\frac{\frac{\partial V(n', k', z')}{\partial n'}}{\frac{\partial V^{cont}(n, k, z)}{\partial n}} - 1\right)}_{<0: \text{ tightness of the financing constraint at date t}} |z\right]$$

The LHS is the difference between the marginal prices of risky bonds and the safe bond, which is negative if there is a positive probability of default. In RHS, the term inside the parenthesis $\frac{\frac{\partial V(n', k', z')}{\partial n'}}{\frac{\partial V^{cont}(n, k, z)}{\partial n}} - 1$ is the gap between the marginal rate of substitution between date-t+1 book value of net worth and date-t liquidity and unity—a relative measurement of the tightness of the financing constraint at date t. This term tends to be negative when the

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firm faces a tight liquidity constraint at date t so that the marginal value of date-t liquidity tends to exceed the marginal date t+1 book value of net worth. Thus, this equation says the firm is willing to issue risky debt if it faces a significant liquidity constraint at date t.

Since working capital is also subject to financial frictions, the labor wedge can be obtained by taking F.O.C. with respect to date-t+1 labor l', which is chosen and financed at date t:

$$\left(w - \frac{\partial F(b', k', l', z)}{\partial l'}\right) \cdot \frac{\partial V^{cont}(n, k, z)}{\partial n} = \mathbb{E}\left[\Lambda' \text{MRPL}' \frac{\partial V(n', k', z')}{\partial n'} | z\right]$$

The LHS is the marginal cost of hiring extra workers. The first term is the real wage rate minus the extra fund raised by increasing employment (with a constant face value of debt), and the second term is the marginal value of date-t liquidity. The RHS is the marginal benefit of having one extra worker: the product of the entrepreneur family's stochastic discounting factor Λ', the marginal revenue product of labor $\text{MRPL}' = \frac{\partial}{\partial l'}[(A'z'\varepsilon'k'^{\alpha}l'^{1-\alpha})^{1-\frac{1}{\theta}}Y'^{\frac{1}{\theta}}]$, and the marginal value of liquidity at date t+1. We can rewrite the first order condition as:

$$\underbrace{w - \frac{\partial F(b', k', l', z)}{\partial l'}}_{\text{marginal cost of hiring worker}} = \mathbb{E}[\Lambda' \text{MRPL}'] + \underbrace{\mathbb{E}[\Lambda' \overbrace{(\frac{\frac{\partial V}{\partial n'}}{\frac{\partial V^{cont}}{\partial n}} - 1)}^{<0:\ \text{tightness of financing constraint}} \text{MRPL}' | z]}_{\text{Labor Wegde}} \quad (27)$$

The first term of RHS is the standard expected discounted marginal revenue product of labor. The second term tends to be negative if the firm issues risky debt with non-zero default probability, following equation 26. When the firm faces a large idiosyncratic uncertainty, the extra funds raised by increasing employment with a constant face value of debt are limited. Therefore, the firm tends to cut employment when it faces a tight financing constraint, partly because it cannot easily adjust fixed capital downward, as we see below.

Finally, let us solve the decision rules for fixed capital investment. Similar to the unconstrained case, let us define the marginal Q as the marginal benefit of investment:

$$Q^m(k'; n, k, z) \equiv \mathbb{E}[\Lambda' \frac{dV(n', k', z')}{dk'} | z] + \frac{\partial F}{\partial k'}$$

$$= \mathbb{E}\left[\Lambda'\left(\underbrace{\frac{\partial V(n', k', z')}{\partial k'}}_{<0:\text{irreversibility punishment}} + \underbrace{\frac{\partial V}{\partial n'} \times \text{MRPK}'}_{>0:\text{liquid wealth}} + \underbrace{\frac{\partial V}{\partial n'} \times (1-\delta)q}_{>0:\text{illiquid wealth}}\right) | z\right] + \frac{\partial F}{\partial k'}$$

$$(28)$$

Compared with the case without financial frictions, having one additional unit of fixed capital increases the current period borrowing by $\frac{\partial F}{\partial k'}$. This term would be zero if the current period borrowing does not involve any default risks. Following the first-order conditions w.r.t k' and the envelop conditions. The optimal choice of fixed capital is:

- buy capital to $k' = k^+$, if $(1 - \delta)k < k^+$

$$Q^m(k^+; n, k, z) = \frac{\partial V^{cont}(n, k, z)}{\partial n} = q(1 + \phi)$$

- sell capital to $k' = k^-$, if $(1 - \delta)k > k^-$

$$Q^m(k^-; n, k, z) = \frac{\partial V^{cont}(n, k, z)}{\partial n} = \eta q(1 + \phi)$$

- maintain capital at $k' = (1 - \delta)k$ if

$$\eta q(1 + \phi) < [Q^m(k^-; n, k, z) < q(1 + \phi)$$

Figure 4 displays the investment policy in a diagram. Similarly, we can define the fixed capital investment wedge for the upward adjustment and downward adjustment separately.

$$\underbrace{q(1 - \mathbb{E}[\Lambda'(1 - \delta)])}_{\text{use cost: upward adjustment}} = \mathbb{E}[\Lambda'\text{MRPK}'] + \underbrace{\mathbb{E}[\Lambda'\,(\overbrace{\frac{\frac{\partial V}{\partial n'}}{\frac{\partial \tilde{V}}{\partial n}}}^{<0:\text{ fin. cons.}} - 1)(\text{MRPK}' + (1 - \delta)q)] + \mathbb{E}[\Lambda'\frac{\frac{\partial V}{\partial k'}}{\frac{\partial V^{cont}}{\partial n}}] + \frac{\frac{\partial F}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}}_{\text{Investment Wedge}}$$

(29)

$$\underbrace{\eta q(1 - \mathbb{E}[\Lambda'(1 - \delta)])}_{\text{use cost: downward adjustment}} = \mathbb{E}[\Lambda'\text{MRPK}'] + \underbrace{\mathbb{E}[\Lambda'\,(\overbrace{\frac{\frac{\partial V}{\partial n'}}{\frac{\partial \tilde{V}}{\partial n}}}^{<0:\text{ fin. cons.}} - 1)(\text{MRPK}' + (1 - \delta)q)] + \mathbb{E}[\Lambda'\frac{\frac{\partial V}{\partial k'}}{\frac{\partial V^{cont}}{\partial n}}] + \frac{\frac{\partial F}{\partial k'}}{\frac{\partial \tilde{V}}{\partial n}}}_{\text{Investment Wedge}}$$

(30)

The LHS is the standard definition of the user cost of fixed capital. The first term in the RHS is the present value of the marginal revenue product of capital. With financial frictions and irreversibility, an investment wedge rises. The first term in the investment

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Figure 4: Decision Rules Among Constrained Firms
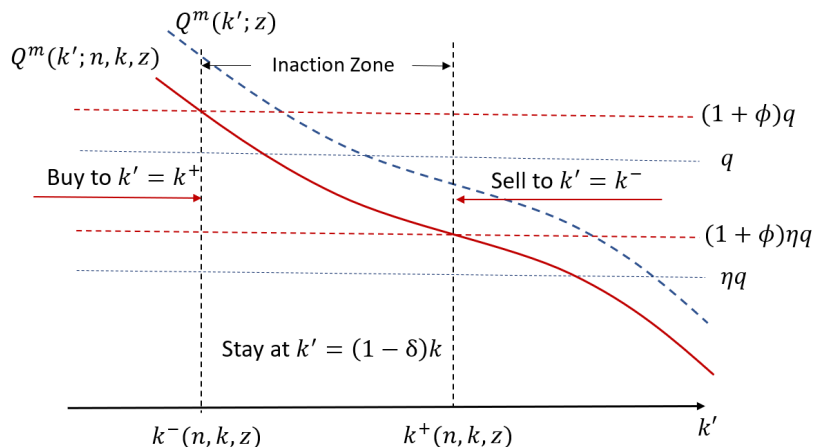
wedge captures the default premium. The second term in the investment wedge is the punishment of having irreversibility, which pushes down the investment level.

Let us compare the decision rules with unconstrained firms or what happens when the financing constraint becomes tighter. First, let us consider the movement of the marginal benefit of investment, which is governed by three forces: (i) Because working capital is subject to financing constraints, insufficient working capital lowers expected MRPK. The second term in (28) decreases, which shifts down $Q^m(n, k, z)$ (ii) Constrained firms are more likely to sell capital under fire sale. As a result, the punishment of irreversibility tends to be higher, and the first term $\frac{\partial V(n',k',z')}{\partial k'}$ in (28) decreases. This shifts down $Q^m(n, k, z)$ (iii) Since firms are financially constrained, $\frac{\partial V(n',k',z')}{\partial n'}$ is larger and this shifts up $Q^m(n, k, z)$. However, a higher expected marginal value of liquid wealth implies a higher current marginal value of liquid wealth. This upward shift will be canceled by the upward shift in $(1 + \phi)q$ when the current borrowing constraint does not bind. When the borrowing constraints bind, the upward shift in $(1 + \phi)q$ will dominate.

As a result, the purchasing threshold k^+ decreases because of the downward shift in Q^m and the upward shift in the shadow price of fixed capital. The two powers discourage firms from investing. Additionally, the selling threshold decreases for the same reason, which implies selling is encouraged compared to the case without financial frictions. For firms with tight financial constraints with a high marginal value of liquid wealth, the selling threshold would be very low and create inefficient liquidation. This extra selling is inefficient since it is triggered by financial frictions. Finally, for the firms in the investment inaction zone, tighter

financial frictions imply a lower MRPK, holding all else equal. For firms outside the inaction zone, tighter financial frictions imply a higher MRPK. We will test these unique predictions from our model in the next subsection.

Next, let us discuss the dynamic properties by considering a firm receives an adverse shock to its level of revenue productivity. First, for the firms in the purchasing zone where Equation (29) is satisfied, the adverse shock shifts Q^m down. The default risks shoot up since the expected net worth is lower, and default thresholds become higher. Additionally, the existence of financial friction makes the punishment of irreversibility $\frac{\partial V}{\partial k}$ even stronger because of the involved default risks. The three forces push the purchasing threshold to the left hand side.

Second, let us consider the firms in the inaction zone, where firms evaluate the RHS in Equation (29) at $k' = (1 - \delta)k$ and find the present value of marginal investment is lower than the upward user cost of capital but higher than the downward user cost of capital. As a result, they would not sell fixed capital immediately. The adverse shock will lower the MRPK and tighten the financing constraint. The tightening financing constraint will also affect the choice of labor through the labor wedge. As a result, firms will find they have too much fixed capital and too little working capital input in the short run. This will additionally decrease MRPK because working capital can't be financed to the optimal level. Firms are expected to lose net worth under this unbalanced and unprofitable production structure. As a result, the default premium further rises. An endogenous feedback loop at the individual level is created. A higher default premium lowers the MRPK, and a lower MRPK pushes up the default premium. Notice that the feedback loop does not exist if we do not have irreversibility because firms can increase MRPK and decrease default risks by selling fixed capital at the same time. As the firm goes into this spiral, the present value of marginal investment will decrease quickly, and it will finally equalize with the downward user cost of capital. At that point, fire sale and inefficient liquidation will be triggered. When firms are close to the default threshold with a high marginal value of liquid wealth. Firms have to sell fixed capital even though it is very costly. In extreme cases, firms have to default due to a low net worth value. Eventually, the firm will face the trade off between a lower default risk with a lower option value by selling fixed capital early and a higher default risk with a higher option value by holding fixed capital stock. This is the critical implication when we combine financial friction and irreversibility together.

Next, let us consider the role of the capital recovery rate in this model. It affects the financing constraint, irreversibility, and the interaction between the two frictions. Firstly,

a lower capital recovery rate implies a lower collateral value, and the credit spread will be higher, conditional on the same default risks and pledgeability of fixed capital. Consequently, a lower capital recovery rate delivers a higher default threshold. Secondly, a Lower capital recovery rate implies a wider investment inaction zone. Thirdly, a lower capital recovery rate implies the firm tends to bear higher default risks and lower MRPK in the investment inaction zone. The above mechanisms imply an economy/sector with a lower capital recovery rate tends to have large distortions in the steady state and be more vulnerable to aggregate shocks.

3.3   Empirical Evidence

Data   We utilize the Annual Survey of Industrial (ASI) Firms in China from 1998 to 2013 to examine empirical predictions from our model. [17] The data contains several advantages. First, it covers all the firms with revenue over 5 million RMB (700K USD) from 1998 to 2010 and all the firms with revenue over 20 million RMB (3M USD) from 2011-2013, leaving a large sample size from 100 thousand to 300 thousand a year. Second, more than 99 percent of firms are not publicly listed firms. As a result, financial frictions are a main obstacle among small and medium-sized firms. Meanwhile, mergers and acquisitions are not the main source of exiting choices for those firms which we do not consider in our model. Therefore, this sample is a good fit for our model. Third, this data set contains variables in both production and balance sheets. This allows us to study investment dynamics and financial frictions together. See more introduction about the data set in Appendix A.

Firm Level Evidence   The first object we examine is the decisions on default. Our model offers three properties of the default threshold. (i) Higher productivity implies lower default risks as stochastic monotonicity holds, (ii) higher net worth implies a lower default risk, and (iii) higher fixed capital is associated with a higher default risk conditional on net worth if and only if downward adjustment solves the maximum dividend payment problem. Our firm-level data set does not carry any information on default, so we use exit decisions as a proxy to measure default risks using panel data where E_{it} is a dummy variable that equals one if the firm does not appear in our sample onward. Fixed capital k_{it} includes assets in equipment, buildings, structures, and other productive capital measured in terms of the book value, deflated by the national price index of fixed-asset investment. The investment i_t is

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[17]2010 data is not available, and not all variables are available in all years. Please see Appendix A for a detailed description

the fixed capital investment defined as the capital stock in the end of this year minus the capital stock in the end of last year plus depreciation in this year, $i_t = k_{i,t+1} - k_{it} + \delta_{it}$. Net worth n_{it} is total assets minus total liabilities. TPFR z_{it} is the residuals from a production function estimation.

We estimate variants of the baseline empirical specification with a linear probability model.

$$E_{it} = \alpha_i + \alpha_{st} + \beta_1 \log n_{it} + \beta_2 \log z_{it} + \beta_3 \log k_{it} + \beta_4 \mathbb{I}\{i_t \leq 0\} \times \log k_{it} + \Gamma' Z_{i,t-1} + e_{it} \quad (31)$$

where α_i is a firm i fixed effect, α_{st} is a year-two digit sector fixed effect, $\mathbb{I}\{i_t \leq 0\}$ is an indicator specifies whether one makes a positive adjustment in fixed capital or not. Table 1 reports the results. Property (i) and (ii) are verified with data since higher productivity or higher net worth is associated with a higher lower exit probability, holding all else equal. Columns 1 and 3 also suggest that higher fixed capital is associated with a lower default probability conditional on productivity and net worth. [18]. However, this effect is heterogeneous depending on the investment actions by looking at the coefficients on interaction term. By looking at β_4 in columns 2 and 4, higher fixed capital implies a higher probability of exit in the next period if and only if firms make a non-positive investment. This is exactly consistent with Proposition 1, which shows higher fixed capital is associated with a higher default threshold conditional on net worth if and only if downward adjustment solves the maximum dividend payment problem in the presence of irreversibility.

The second object we examine is how the interaction between irreversibility and financial friction affects capital misallocation. Normally, tighter financial constraint is associated with a higher MRPK, and MRPK is particularly high for firms with higher productivity, but not enough net worth can be pledged as collateral. As a result, financial frictions dampen the speed of accumulating wealth in the transition in response to a persistent positive shock. However, asset demand goes down in response to negative shocks, and financial constraint is likely to be loosened and would not have much effect on investment policies. Interstingly, adding irreversibility makes things more sophisticated. According to Figure 4, for firms outside the investment inaction zone, a tighter financial constraint shifts the shadow price of capital $(1 + \phi)q$ or $(1 + \phi)\eta q$ up. Resulting in a lower level of fixed capital and higher

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[18]This is not the prediction from our model. To rationalize this fact, one reason is the definition of net worth in our model is not fully consistent with what in the data. In the model, the definition of net worth is assets plus revenue minus liability, while in the data, net worth is assets minus liability. Therefore, a higher fixed capital implies a higher potential revenue and higher net worth in the model conditional on the same net worth in the data.

Table 1

	(1) Exit	(2) Exit	(3) Exit	(4) Exit
Log(Net Worth)× D(Net Worth>0)	-0.010***	-0.010***	-0.010***	-0.010***
	(0.000)	(0.000)	(0.000)	(0.000)
Log(-Net Worth)× D(Net Worth<0)	0.008***	0.008***	0.020***	0.020***
	(0.000)	(0.000)	(0.000)	(0.000)
Log(TFPR)	-0.011***	-0.011***	-0.016***	-0.016***
	(0.001)	(0.001)	(0.001)	(0.001)
Log(Fixed Capital)	-0.014***	-0.011***	-0.008***	-0.007***
	(0.000)	(0.000)	(0.000)	(0.000)
D(Investment≤0)× Log(Fixed Capital)		**0.016*****		**0.018*** **
		(0.000)		(0.000)
Constant	0.309***	0.299***	0.230***	0.213***
	(0.001)	(0.001)	(0.004)	(0.004)
Year × Sector FE	Yes	Yes	Yes	Yes
Firm FE	No	No	Yes	Yes
Observations	1932468	1932468	1779519	1779519
R-Square	0.062	0.064	0.370	0.373

Notes: Results from estimating (31). Fixed capital k_{it} includes assets in equipment, buildings, structures, and other productive capital measured in terms of the book value. The investment is fixed capital investment $i_t = k_{i,t+1} - k_{it} + \delta_{it}$. Net worth is total assets minus total liabilities. TPFR is residuals from a production function estimation. Data source: Chinese Annual Survey of Industrial Firms, 1998-2007. Standard errors are clustered at the firm level in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

marginal benefit of investment associated with higher expected MRPK. In contrast, for firms in the investment inaction zone, tighter financial constraints would not affect the choice of the level of fixed capital. However, a tighter financing constraint would shift the $Q^m(n, k, z)$ down as working capital can't be invested to the optimal level. The lower marginal benefit of investment is associated with a lower expected MPRK. As a result, tighter financing constraint is associated with higher MRPK outside the investment inaction one and associated with lower MRPK inside the investment inaction zone.

To test this unique model's implications, we estimate variants of the baseline empirical specification:

$$\log y_{it} = \alpha_i + \alpha_{st} + \beta_1 \log x_{it} + \beta_2 \mathbb{I}\{i_{it} = 0\} \times \log x_{it} + \Gamma' Z_{i,t-1} + e_{it} \qquad (32)$$

where α_i is a firm i fixed effect, α_{st} is a year-two digit sector fixed effect, $\mathbb{I}\{i_t = 0\}$ is an indicator specifies whether one makes a positive adjustment in fixed capital or not. we use ARPK y_{it} defined as value-added over fixed capital to measure MRPK. The tightness of financing constraint is proxied by two variables $x_{it} \in \{\text{leverage ratio}, \text{borrowing rate}\}$. The leverage ratio is defined as total liabilities over total assets, and the borrowing rate is defined as total interest payment over total liabilities.

Table 2

	(1) Log(ARPK)	(2) Log(ARPK)
Log(Debt/Asset)	0.054***	
	(0.002)	
D(Investment=0)× Log(Debt/Asset)	**-0.074*****	
	(0.002)	
Log (Borrowing Rate)		0.033***
		(0.001)
D(Investment=0) × Log(Borrowing Rate)		**-0.017*****
		(0.001)
Constant	0.160***	0.072***
	(0.001)	(0.004)
Year × Sector FE	Yes	Yes
Firm FE	Yes	Yes
Observations	2875653	2140368
R-Square	0.839	0.855

Notes: Results from estimating (32). α_i is a firm i fixed effect, α_{st} is a year-two digit sector fixed effect, $\mathbb{I}\{i_t = 0\}$ is an indicator specifies whether one makes a positive adjustment in fixed capital or not. ARPK y_{it} is defined as value-added over fixed capital to measure MRPK. The tightness of financing constraint is proxied by two variables $x_{it} \in \{\text{leverage ratio, borrowing rate}\}$. The leverage ratio is defined as total liabilities over total assets, and the borrowing rate is defined as total interest payment over total liabilities. Data source: Chinese Annual Survey of Industrial Firms, 1998-2007. Standard errors are clustered at the firm level in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2 reports the results. As predicted in the model, ARPK is positively correlated with the leverage ratio and borrowing rate for the firms that make active capital adjustments. In contrast, APRK is negatively correlated with the leverage ratio for firms that are in the investment inaction zone. Though the results on borrowing rate are not strong enough to reverse the positive correlation, it dampens the positive correlation, and the empirical results are generally in line with the properties of steady-state decision rules implied by the interaction between financial frictions and irreversibility.

Sector Level Evidence Up to now, we have shown the firm-level empirical evidence, which is in line with the model properties coming from the interaction between financial frictions and partial irreversibility in steady-state. These results support the validity of the model we built. Back to our original question, we want to understand the role of irreversibility and financial frictions in propagating aggregate shocks. Ideally, if there are various economies with different capital recovery rates in the lab, it is straightforward to examine the dynamics with respect to various economies in response to aggregate shocks. Although we do not have such a research lab in economics, we might be able to utilize the sectoral differences in capital recovery rates and examine the differences in sectoral dynamics in recessions. Fortunately,

Kermani and Ma (2023) computes sector-specific fixed capital recovery rate in the US using bankruptcy documentation. Though I do not have the same measurement using Chinese data, I argue this capital recovery rate is a sector-specific feature, and the variance should be informative in another economy. To examine the relationship between capital recovery rate and dynamics in response to aggregate shocks, we estimate variants of the following empirical specification:

$$y_{it} = \alpha_i + Q(t, t^2) + \beta_1 D_t + \beta_2 D_t \times \eta_i + \beta_3 D_t \times s_i + e_{it} \tag{33}$$

where α_i is a two-digit sector-level fixed effect. $Q(t, t^2)$ is a quadratic form that controls the non-linear aggregate trends. $y_{it} \in \{\text{St.d.(log ARPK)}, \log \text{Output}\}$ is two sector-year level outcomes. We use standard deviation in ARPK within a sector to measure the misallocation and sector-level output to measure aggregate impacts. D_t is a dummy variable that equals one in the years of 2008 and 2009, the only recession in the span of our data coverage. η_i is the sector-specific capital recovery rate. s_i is the sector-level export share defined as export value over output before the recession in 2007. This measures the degree of demand shock in the recession.

Table 3 reports the results. From Panel A, the standard deviation in $\log ARPK$ rises by 3 percent on average in the recession relative to the trend, suggesting misallocation rises in the recession. However, this relationship is heterogeneous and heavily depends on the capital recovery rate. An 0.1 increase in capital recovery rate is associated with a 1.3 percent decline in dispersion in ARPK in the recession. In column 3, sectors with higher export share pre-recession are also associated with a higher dispersion. This result suggests that a higher demand shock is associated with a higher misallocation. Panel B conveys a similar set of information. Relative to the trend, the average output loss is around 7 percent in the two recession years. This loss is heterogeneous across sectors. An 0.1 increase in capital recovery rate is associated with a 9 percent gain in output in the recession. Sectors with higher export share pre-recession are also associated with a higher output loss in the recession, though not statistically significant.

To summarize, the results in Table 3 suggest a significant role of irreversibility in propagating aggregate shocks, partially through the misallocation channel. The aggregate shock we consider here in China is likely to be a first-moment demand shock rather than financial shocks or uncertainty shocks. In the quantitative exercises, we will simulate an economy with two sectors to connect the empirical evidence we found here.

Table 3

	(1)	(2)	(3)	(4)
Panel A - Dep. Var. St.d. (log ARPK)				
D(Recession=1)	0.032***	0.076***	0.020***	0.063***
	(0.005)	(0.019)	(0.008)	(0.021)
D(Recession=1)×Recovery Rate		-0.128**		-0.117**
		(0.053)		(0.053)
D(Recession=1)×Export Share			0.050*	0.043
			(0.026)	(0.027)
Constant	1,748.759***	1,748.759***	1,748.759***	1,748.759***
	(341.223)	(339.110)	(340.109)	(338.419)
Observations	420	420	420	420
R-Square	0.759	0.763	0.761	0.764
Panel B - Dep. Var. (log Output)				
D(Recession=1)	-0.070***	-0.377***	-0.035***	-0.345***
	(0.029)	(0.108)	(0.045)	(0.117)
D(Recession=1)×Recovery Rate		0.884***		0.859**
		(0.299)		(0.302)
D(Recession=1)×Export Share			-0.156	-0.103
			(0.151)	(0.151)
Constant	-1,035.363	-1,035.363	-1,035.363	-1,035.363
	(1,943.420)	(1,924.397)	(1,943.276)	(1,925.731)
Observations	420	420	420	420
R-Square	0.979	0.979	0.979	0.979

Notes: The table reports results from estimating 33. where α_i is a two-digit sector-level fixed effect. $Q(t, t^2)$ is a quadratic form that controls the non-linear common trend. $y_{it} \in \{$St.d.(log ARPK), log Output$\}$ is two sector-year level outcomes. D_t is a dummy variable that equals one in the years of 2008 and 2009. η_i is the sector-specific capital recovery rate taken from Kermani and Ma (2023). s_i is the sector-level export share defined as export value over output in 2007. Standard errors are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Aggregate Level Evidence   In this paragraph, we present some empirical evidence about the cyclicality of misallocation. We find that within-sector dispersion in MRPK is counter-cyclical in both China and the U.S. in the manufacturing sector. To measure the cyclicality of misallocation in China, I compute the standard deviation of ARPK within 4-digit sector and take a weighted average by sector share in output. To measure the cyclicality of misallocation in the U.S., I utilize a data set constructed in Bils et al. (2021) where they provide a measurement error-adjusted annual sequence of allocative efficiency in the US manufacturing sector. The allocative efficiency is defined as the current output divided by the potential output once the marginal revenue of inputs is equalized.

In Figure 5 (a), I plot the detrend standard deviation of ARPK (removing linear trend)

versus GDP growth rate in China from 2004 to 2012. These two variables are negatively correlated. In Figure 5 (b), I plot the HP-filtered allocative efficiency versus GDP in the U.S. manufacturing sector. In the past three recessions in the U.S., the allocative efficiency went down. The correlation between the cyclical component of allocative efficiency and GDP is 0.32. The allocative efficiency also seems a bit lagged. Though it is hard to identify what kind of shocks drive the business cycles, misallocation is suggested to have a role in propagating the aggregate shocks.

Figure 5: Counter Cyclical Dispersion in MRPK

(a) China   (b) U.S.
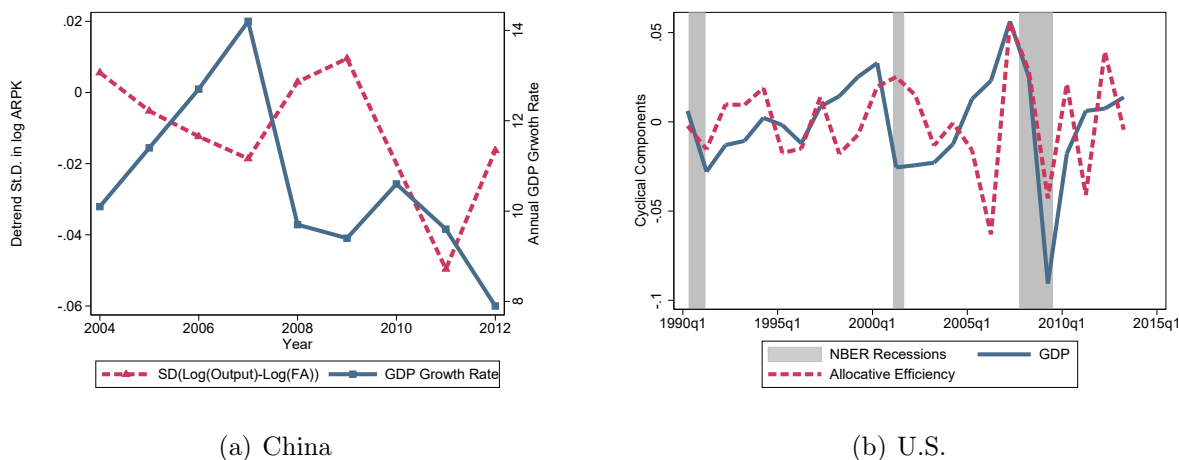
Notes: Panel A: Data Source: St.D. of ARPK is computed from the Annual Survey of Industrial firms 2004-2012 (no 2010) in China within the 4-digit sector at annual frequency.
Panel B: Log deviations from HP trend (smoothing parameter $\lambda = 6.25$) of (i) Allocation efficiency data is from (Bils et al. (2021)). (ii) US real GDP, manufacturing sector. Annual frequency. Corr(allocative efficiency, GDP)=0.32

4 Calibration

In this section, we calibrate the model we describe in section 2 to the Chinese economy. The calibration process is conducted in two steps. We first select a set of externally calibrated parameters from the literature or our estimation from the micro-level data. Secondly, we discipline our model by choosing a set of internally calibrated parameters to match a set of simulated moments with empirical moments in the data. Then, we examine some untargeted moments to verify the calibrated model is a validated lab that represents the distribution and behaviors of the economy in the steady-state.

4.1 Externally Calibrated Parameters

Table 4 presents the choices of externally calibrated parameters. The quarterly discounting factor is chosen as 0.99, implying an annual risk-free rate of 4 percent in the steady state. As we choose GHH (Greenwood et al. (1988)) preferences, the elasticity of labor supply is independent of the marginal utility of consumption. We choose the Frisch elasticity of labor supply to be 1. Though the number is greater than some of the early micro estimations surveyed by Blundell and MaCurdy (1999), it is consistent with compensated elasticities at the macro level Keane and Rogerson (2015). We choose the measure of the representative entrepreneur family to be 0.5, implying half of the population in China is hand-to-mouth. Though it is an arbitrary assumption, it fits the fact that a large share of households are still relatively poor with little financial assets and net worth in China.

Capital share and elasticity of substitution are chosen from the production estimation in (3); please see Appendix A for the results. We choose the quarterly depreciation rate to be 0.025, which is standard in the literature. Notice that due to the irreversibility, the macro aggregate depreciation rate $\tilde{\delta}$ is larger than the micro depreciation rate δ, which implies an annual I/K ratio well above 0.1 in the steady state. The aggregate adjustment cost of capital goods is governed by Ψ. We choose it to be 2, implying a one percent change in I/K is associated with a 0.5 percent change in the price of capital. The idiosyncratic process of revenue productivity is estimated from the residual dynamics of production estimation using the Annual Survey of Industrial Firms in China. Compared to the literature. the micro-level uncertainty is higher in China than in advanced economies like the U.S.

4.2 Internally Calibrated Parameters

In the second step, we choose a set of parameters to match a set of empirical moments jointly. Though some of the parameters are more informative to a particular moment than others, the model is nonlinear, and one-on-one mapping is not possible. Table 5 reports the values of internally calibrated parameters, and table 6 reports the comparison between the targeted model simulated moments and the empirical moments. First, the fixed operation cost shifts the default threshold directly following (12). A higher fixed operation cost implies a higher default threshold and higher default density in the steady state. Our empirical default rate is the fraction of non-performing loans in China reported by the China Banking Regulatory Commission. We choose the quarterly exogenous exit rate to be 0.02, combined with the annual default rate of 0.04, contributing to a total annual exit rate of 0.12, which is observed

Table 4: Summary of Externally Calibrated Parameters

Parameters	Description/Sources	Value
Households		
β	Discounting factor	0.99
ν	Inverse of elasticity of labor supply	1
M	Measure of entrepreneur family	0.5
Firms		
α	Capital share	0.30
δ	Depreciation rate	0.025
θ	Elasticity of substitution	5.0
Φ	Elasticity of q to I/K	2
Idiosyncratic Processes		
ρ	From micro estimates	0.75
σ_ν	From micro estimates	0.08
σ_ε	From micro estimates	0.18
Policies		
λ^G	Elasticity of fiscal policy	1
λ^{τ^D}	Elasticity of credit policy	4

from our calculation from the Annual Survey of Industrial Firms in China. Next, we assume exiting firms are replaced by newborn firms with the same initial equity n_0. Newborn firms are smaller on average and take time to grow. As a result, the default cost is persistent and is governed by the size of newborn firms. We choose initial equity n_0 to match the ratio of average net worth for young firms with ages less than 3 years to the average net worth of all the firms. The higher n_0 is, the higher this ratio would be in the simulated data. Finally, we identify the capital recovery rate to match the ratio of firms that make zero fixed capital investments. In the firm-level investment data, we allow for small capital adjustment when we define investment inaction zone. We classify firms with absolute annual empirical investment rates less than 0.01 as inactive firms, and we find the share of inactive firms is 0.21. In the model, a lower capital recovery rate implies a wider investment inaction zone and a higher density of firms that make zero investments in fixed capital. The capital recovery rate also determines the collateral value. It is informative to default thresholds, default density in the steady state, and credit spread in the model. Therefore, the default rate is pinned down jointly by the fixed operation cost and capital recovery rate. The calibrated capital recovery rate is 0.43, close to what is suggested in Kermani and Ma (2023). Finally, $\overline{G}$ is chosen to match the government expenditure ratio of 0.25.

Table 5: Summary of Internally Calibrated Parameters

Parameters	Description	Value
ξ	Operating cost	0.15
π_d	Exogenous exit rate	0.02
n_0	Initial equity	0.18
η	Recovery rate	0.43
$\overline{G}$	Government expenditure	0.42

Table 6: Calibration Target and Model Fit

Moments	Description	Data	Model
$\mathbb{E}[\text{default rate}]$	Annual default rate	0.04	0.04
$\mathbb{E}[\text{Exit rate}]$	Annual exit rate	0.12	0.12
$\mathbb{E}[\frac{n(age \leq 3 \text{years})}{\overline{n}}]$	Ratio of net worth at age 1-3 to mean	0.49	0.48
$\mathbb{E}[\text{Investment}=0]$	Ratio of annual inactive firms	0.21	0.21
$\overline{G}/Y = 0.25$	Ratio of government expenditure	0.25	0.25

5 Quantitative Results

In this section, with the calibrated model, we examine the macroeconomic implications of the interaction between financial frictions and the partial irreversibility of fixed capital by simulating the transition dynamics in response to aggregate shocks under variants of the model.

5.1 Aggregate Responses to Productivity Shock

We first study the transition dynamics in response to an ex-ante unexpected but ex-post fully anticipated aggregate productivity shock. To assess the amplification effect from the interaction between financial frictions and irreversibility, I simulate the same shock in three economies. The baseline case features both financial friction and irreversibility. The economy only features irreversibility where financial frictions are shut down by assuming external debt payment is fully committed and equity issuance is not associated with any additional cost. And the economy only features financial frictions where I set $\eta = 1$. The TFP in the steady state is $\overline{A} = 1$, the shock I consider is:

$$\log A_{t+1} = \rho_A \log A_t + (1 - \rho_A) \log \overline{A} + \varepsilon_t^A$$

where the economy starts from the steady state and TFP shock hits at t_0 unexpectedly while $\varepsilon^A_{t_0} = -0.02$ and $\varepsilon^A_{t \neq t_0} = 0$. The persistence of the shock is $\rho_A = 0.7$. The definition of the equilibrium in the transition path is defined in Appendix B.

Proposition 4 *Along the transition dynamics with $\varepsilon^A_{t_0} < 0$, the default threshold is higher than in the steady state*

$$\{\underline{n}_t(k,z)\}^\infty_{t=t_0} > \underline{n}(k,z)$$

and the threshold specifies the default threshold dependency on k is lower

$$\{\underline{k}_t(z)\}^\infty_{t=t_0} < \underline{k}(z)$$

Proof: See Appendix C.

Figure 6 displays the results. The baseline simulation shows a sizeable amplification effect is generated from the interaction between financial frictions and irreversibility compared to a model with only financial frictions or real frictions. A 2 percent TFP shock leads to a maximum of 3.9 percent and a more persistent decline in output. Since inputs are predetermined, the drop in output when the aggregate shock hits comes from the decline in TFP and extra default. Surviving firms choose investment decisions.

As illustrated in Figure 7, negative TFP shock lowers the marginal benefit of investment and pushes a more significant density of firms into the investment inaction zone unexpectedly from the left-hand side. As the shock is temporary, selling the capital immediately is very costly. The financing constraint on working capital becomes tighter for three reasons. First, a higher fixed capital level increases the demand for working capital. Second, default risks rise as the expected cash flow is lower. Third, a lower capital price decreases the collateral value, which drives up the credit spread conditional on default risks. As a result, working capital can't be financed to an optimal level. Consequently, misallocation, measured by the standard deviation in ARPK, increases by 2 percent. Then, if the financing constraint becomes even tighter with a high marginal value of liquid wealth, a fire sale is triggered even though the capital price is lower. In extreme cases, firms have to default. As shown in Proposition 4, the default zone expands along the transition path, and a larger density of firms into the default zone. Since inputs are pre-determined, the output should drop by 2 percent if there is no extra default in the initial period that the shock hits. Extra output loss in that initial period is only associated with extra default. Indeed, default density increases by over 20 percent in the baseline simulation but decays quickly as there are no additional unexpected

adverse shocks. Inefficient liquidation comes from default and fire sale, which pushes down the capital price, and the lower capital price leads to more inefficient liquidation. Since exiting firms will be replaced by small firms which take time to grow, the effect of default on output and labor demand will accumulate and be persistent. Liquidating firms also take time to restock fixed capital. The process is lengthy because irreversibility makes them cautious, and financial frictions make investments costly when the net worth is low.

Labor demand drops for two reasons: the negative TFP shock and the tightening financing constraint on working capital. As a result, the wage rate goes down, and the labor supply shrinks proportionally since there is no wealth effect with GHH preference. Hand-to-month workers can't smooth their consumption, and therefore, the demand for consumption drops by more. Aggregate output is dragged down by extra default and misallocation in addition to TFP shocks. It is also affected by the demand side since hand-to-month agents consume less. This lower aggregate demand will enter the aggregate revenue productivity according to equation (3). As a result, it works as an additional negative quasi-TFP shock and constitutes an endogenous feedback loop. Notice that amplification will not exist if Y_t moves with A_t at the same pace. But a more profound drop in Y_t relative to A_t will be amplified endogenously.

Then let us examine the economy with no financial frictions. The default channel is shut down entirely without financial friction. There would also be no misallocation. The existence of real frictions will lead to a slightly larger dispersion in ARPk because a larger share of firms are pushed into the investment inaction zone. However, the decline in capital price and interest rate pushes down the purchasing threshold and stimulates investment in the general equilibrium. As a result, the investment inaction zone would not move by much. Furthermore, firms in the inaction zone can always finance enough working capital, and therefore, dispersion in ARPK would not increase by much. There would be no extra power to shift down the labor demand relative to a standard RBC model. As a result, output only decreases by precisely 2 percent when the shock hits, and there is no significant amplification effect along the transition path.

In a model with only financial frictions, an inaction zone does not exist. Firms would shrink immediately in response to negative TFP shock. The default threshold, only depending on net worth now, will increase, but default density only rises modestly without irreversibility. The dispersion in ARPK even goes down slightly when the aggregate shock just hits because the decline in fixed capital demand outweighs the decline in collateral value. This is because the demand for inputs is a static choice without adjustment cost, while the

capital price is forward looking. Along the transition path, firms are losing net worth, and the demand for fixed capital rebounds. Consequently, the decline in collateral value gradually overweighs the decline in capital demand. Misallocation picks up in the middle and makes the recession more persistent than the economy with only real frictions.

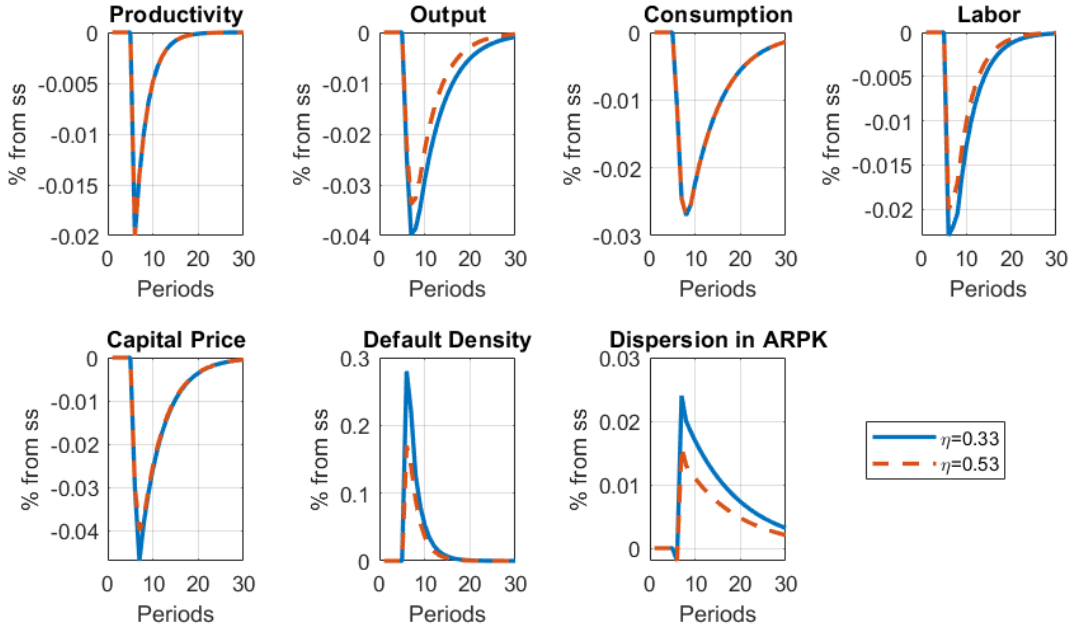**Figure 6:** IRFs to TFP Shock
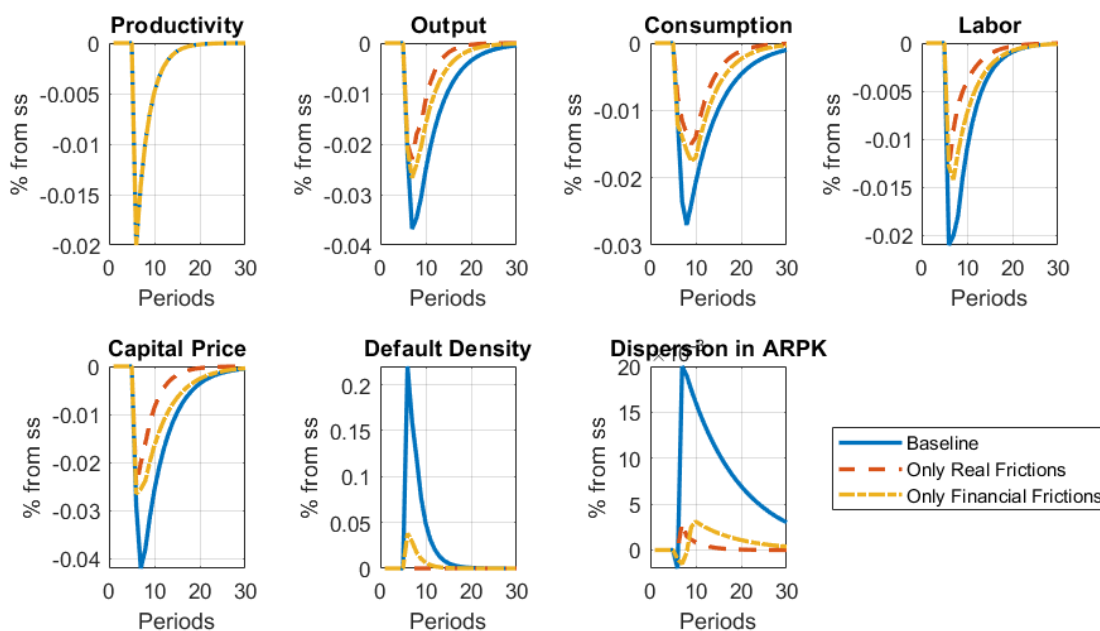
Notes: Aggregate impulse responses to 2 percent negative TFP shock which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6.

Figure 7
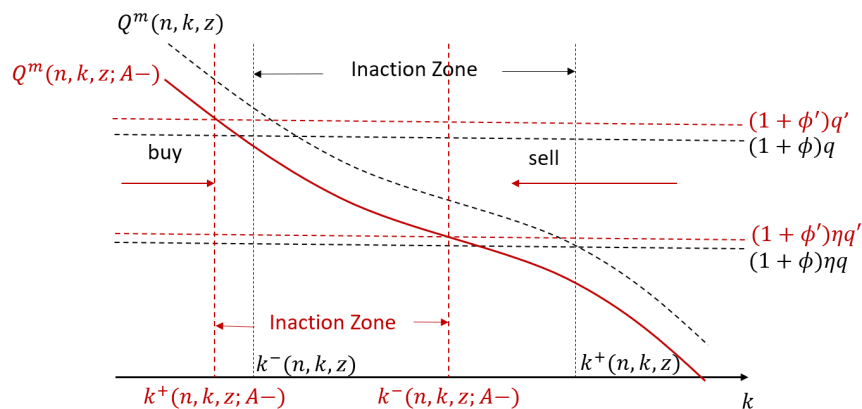
Figure 8: IRFs to TFP Shock: Two Sector Model

Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6.

Multisector Model   Motivated by the regression results in Table 3, we modify the model with two sectors. We simply assume that intermediate firms are split into two sectors with different capital recovery rates, 0.33 and 0.53, respectively. All else is untouched compared with the baseline model. The simulation results show the sector with a lower capital recovery rate experiences a deeper and more persistent decline in output with a larger amplification effect in response to the same TFP shock. The sector with a lower capital recovery rate also experiences a smaller spike in default probability and a lower increase in dispersion in ARPK. As firms in the sector with lower capital recovery rates face tighter financing constraints and liquidate more aggressively, there is also an associated deeper drop in labor demand. Our model simulation results are generally in line with the regression results.

5.2   Counter-Cyclical Fiscal Policy

To mitigate the recession, we consider a countercyclical fiscal policy. In contrast to New Keynesian models where firms face sticky prices, firms face a combination of "sticky quantity" and financial frictions in our model. As the adverse shock shifts the demand curve inward,

sticky quantity limits the firms' ability to adjust the capital stock and exacerbates financial frictions. A higher aggregate demand can push the demand curve outward, release firms from the investment inaction zone, and restore the efficient positions of fixed capital holding. Financing constraints on working capital will be relaxed endogenously due to higher expected cash flow. To boost the aggregate demand during the recession, we consider the fiscal policy rule specified by the following equation :

$$\log G_t = \log \overline{G} - \lambda^G (\log Y_{t-1} - \log \overline{Y})$$

where a 1 percent downward deviation in GDP relative to steady state at $t-1$ leads to a (λ^G) percent upward deviation in government expenditure relative to steady state at $t+1$. Figure 9 displays the comparison of IRFs between the baseline economy where government expenditure is constant and the economy where countercyclical fiscal policy is active with $\lambda^G = 1$. By boosting aggregate demand, the economy experiences a lower decline in output, capital price, and a smaller rise in default density, as well as dispersion in ARPK. As the financing constraints on working capital become looser, labor demand and the wage rate are boosted together. Higher wage bills translate into higher consumption demand with a large proportion of hand-to-mouth agents, which delivers another upward push to aggregate demand. Notice that even though the government does not increase expenditure in the period when the negative shock hits because the fiscal policy is lagged for one period. It effectively lowers the default density at $t = t_0$ because the bank and firms are forward-looking if the policy is fully committed.

Optimal Fiscal Policy   As shown in Figure 9, counter-cyclical fiscal policy is effective in mitigating the recession, but it is associated with higher government expenditure, which is financed by lump sum tax on the entrepreneur family. As we assume that government expenditure does not affect private agents' utility level, the impact of the credit policy on welfare is ambiguous. Although our model does not contain recurrent aggregate shocks, we can study the optimal fiscal policy in the transition path with full policy commitment in response to a particular type of exogenous shock. We study the optimal fiscal policy within the class of the policy rules we specify with various λ^G. We assume the economy goes back to the original steady state in 10 years (40 periods) and focus on the consumption equivalent welfare gains along the transition path. Though it is not the most ideal environment to study optimal fiscal policy, our positive exercises can recover the fundamental mechanisms in this economy.
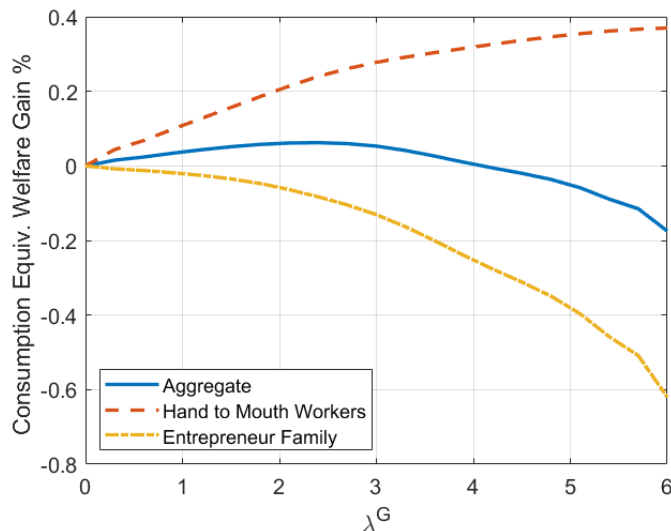
Figure 9: IRFs to TFP Shock: Fiscal Policy
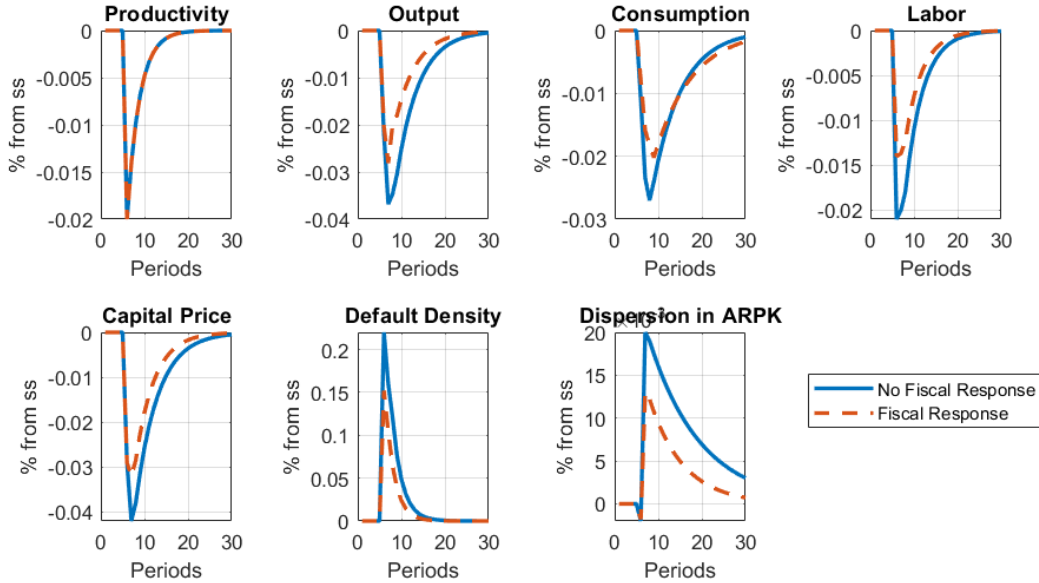
Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6. The solid lines depict the IRFs in the baseline model with no active fiscal policy, and the dashed lines depict the IRFs in the model with counter-cyclical fiscal policy as described in the text. $\lambda^G = 1$

As Hand-to-Month workers do not have access to the financial market to smooth their consumption, a drop in labor demand leads to a relatively significant welfare loss. Higher government expenditure pushes up aggregate demand and improves supply-side efficiency endogenously. Labor demand is higher, resulting from less misallocation, inefficient liquidation, and default. Since Hand-to-Month workers do not pay taxes, the higher λ^G is, the higher welfare gain they get. As the improvement in allocative efficiency is diminishing, The welfare gain of the representative worker family diminishes with λ^G.

In contrast, the entrepreneurs' family bears the cost of financing government expenditures. The direct effect is it would crowd out their consumption C^R. However, they also gain because the intermediate firms' value and labor demand would improve as the government purchases more goods, which crowds in their consumption. The relative strength between the two powers depends on how many firms can be saved from the default zone and inefficient liquidation. The quantitative results in Figure 10 show that the crowding out effect is increasing with λ^G. When the fiscal policy is not so aggressive, a slight increase in government expenditure can generate a more potent impact on the decline in default probability and allocative efficiency. As a result, the crowding-out effect is not that strong, and the aggregate

Figure 10: Consumption Equivalent Welfare Gains under Fiscal Policy

Notes: Consumption equivalent welfare gains under various λ^G along the transition path in responses to 2 percent negative TFP shock, which decays at a rate of 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state. The solid line presents welfare gains for the aggregate economy with equal weight, the dashed line presents welfare gains of the Hand-to-Mouth workers, and the dash-dot line presents welfare gains of the entrepreneur family.

welfare gains are positive. When the fiscal policy is already very aggressive, an increase in government expenditure can't improve the supply-side efficiency, and the crowding-out effect dominates. We find the optimal fiscal policy can deliver a 0.06% increase in consumption equivalent welfare along the transition path with $\lambda^G = 2.1$. It implies that government expenditure should increase by 2.1% in response to a 1 % decline in GDP. The welfare gain is modest in our calculation, but it is still positive even if government expenditure does not enter the utility function.

5.3 Counter-Cyclical Credit Policy

To model credit and macroprudential policies that control credit supply, we assume the government will impose a tax τ^D on the bank when it sells seized capital on the secondary market; the tax revenue will be transferred back to the government in a lump sum way. [19] Government can also choose $\tau^D \in [0,1]$ to control credit supply where the policy rule can be specified as:

$$\tau_t^D = \max\{\overline{\tau^D} + \lambda^{\tau^D}(\log Y_{t-1} - \log \overline{Y}), 0\}$$

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[19]This setup is identical to the one where the tax revenue is transferred back to the household if Ricardian Equivalence holds.

Figure 11: IRFs to TFP Shock: Credit Policy
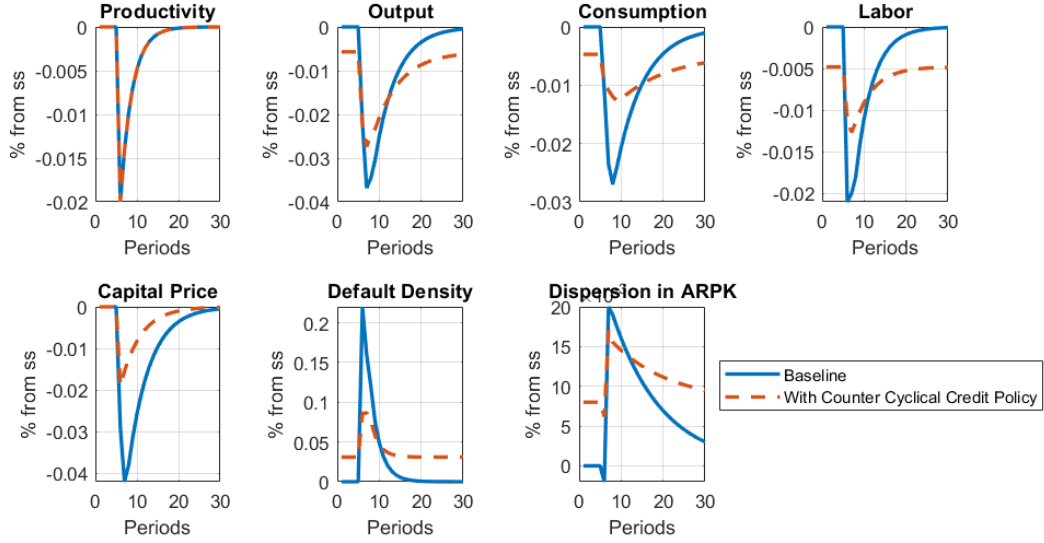
Notes: Aggregate impulse responses to 2 percent negative TFP shock, which decays at a rate 0.7. Computed as the perfect foresight transition in response to a series of unexpected shocks starting from a steady state at t=6. The solid lines depict the IRFs in the baseline model with no active credit policy, and the dashed lines depict the IRFs in the model with counter-cyclical credit policy as described in the text. $\lambda^{\tau^D} = 4$.

where the government relaxes credit supply when the output is lower than the steady-state level but faces an upper bound because the subsidy is not allowed. In Figure 11, we compare two scenarios: the baseline case where the credit policy is relaxed at all time $\tau^D = 0$, and a tighter credit policy in the steady state $\tau^D = 0.2$ with a countercyclical credit policy that loosens the credit constraint in the recession following the rule we specify.

We find that the economy with tighter credit constraints is associated with steady-state efficiency loss as the credit spreads are higher conditional on the same default risks. However, along the transition path, the economy with a tighter credit constraint in the steady state suffers less severe debt overhang problems and has a buffer where credit policy can be relaxed. As a result, the recession was less severe with a counter-cyclical credit policy. The policymakers face a trade-off between steady-state efficiency loss and a more resilient economy in response to adverse shocks.

6 Conclusion

In this article, we argue that the interaction between financial frictions and the partial irreversibility of fixed capital can propagate the first-moment aggregate shocks with a sizeable

amplification effect as the two frictions reinforce each other. In recessions, firms tend to reduce fixed capital as well as working capital if there are no frictions. However, irreversibility makes fixed capital costly to adjust downward. This tightens financing constraints on working capital. The marginal benefit of investment decreases as expected MRPK goes down, and the marginal value of liquidity picks up. Higher default risks and lower expected MRPK create an endogenous feedback loop at the individual level. Constrained firms experience misallocation, inefficient liquidation, and default as the financial constraint becomes tighter gradually. With monopolistic competition, lower aggregate demand further decreases revenue productivity and creates an endogenous feedback loop at the aggregate level with an amplification effect. This result implies that micro-level frictions, the interaction between financial frictions and partial irreversibility, heterogeneity, financial heterogeneity, and portfolio choice are essential to understanding the dynamics of aggregate variables.

Our policy recipe is simple and intuitive. Higher aggregate demand releases firms from the investment inaction zone and relaxes financing constraints endogenously. We find a countercyclical fiscal policy is beneficial, and modest welfare gains are achievable as hand-to-mouth workers gain on higher labor demand and entrepreneurs gain on more efficient production and less inefficient liquidation. In contrast to sticky price models, where the policy recipe's direction is opposite in response to a negative productivity shock or demand shock, our model suggests the policy in the same direction, possibly more intuitive to policymakers' choices.

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